Stochastic Subspace Projection Schemes for Solving Random Blisk Mistuning Problems in Jet Engines

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Objective of this Presentation

To discuss the application of a powerful stochastic subspace projection scheme for solving mistuning problems in bladed-disks via reduced-order modeling (ROM).

The proposed stochastic subspace projection scheme called the Stochastic Perturbation Matrix (SPM) approach provides an efficient tool for accurately solving large (and small) random mistuning problems, for both LO and HO system modes.

Content Presentation

1. Computational Stochastic Mechanics Issues
2. Stochastic Perturbation Matrix Subspace Projection
3. The 72 Blade Compressor Blisk Example
4. Concluding Remarks

1. Computational Stochastic Mechanics Issues

Two Major Stochastic Modeling Aspects:

1. Develop ***Accurate Stochastic Approximation Models*** for High-Complexity Behavior Given Sample Dataset (Data-*based Stochastic ROM*)
   - Global and Local Accuracy in Statistical Data Space – *ROM in Data Space*
   - For both *System Inputs* and Outputs

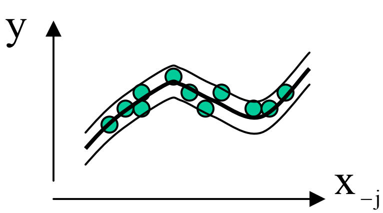
2. Develop ***Fast Stochastic Simulation Models*** Given the Physics (PDE) and Stochastic Inputs (*Physics-based Stochastic ROM*)
   - Global and Local Accuracy in Physical Space – *ROM in Physical Space*
   - For *System Outputs*

3. Combine the Statistics-based and Physics-based Stochastic ROMs

Statistics-based Stochastic ROM

Second-Order (SO) Approximation of Stochastic Fields

Explicit Formulation: Using function approximation via nonlinear regression



$$y|\mathbf{x} = r(\mathbf{x}) + \varepsilon(\mathbf{x})$$

$$E[y|\mathbf{x}] = r(\mathbf{x})$$

--- Least-square fitting (y is explicit)

Stochasticity defined by second-order random vector/field of residuals

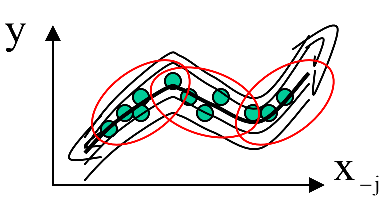
Limited

Convergence: Minimizing Mean-Square Error (in Mean-Square sense)

Causal relationship

High-Order (HO) Approximation of Stochastic Fields

Implicit Formulation: Using joint PDF estimation of $z = [y, \mathbf{x}]^T$



$$f(y|\mathbf{x}) = \frac{f(y,\mathbf{x})}{f(\mathbf{x})} =$$

$$E[y|\mathbf{x}] = \int_{-\infty}^{\infty} y f(y|\mathbf{x}) dy$$

--- Stochastic Networks (y is implicit)

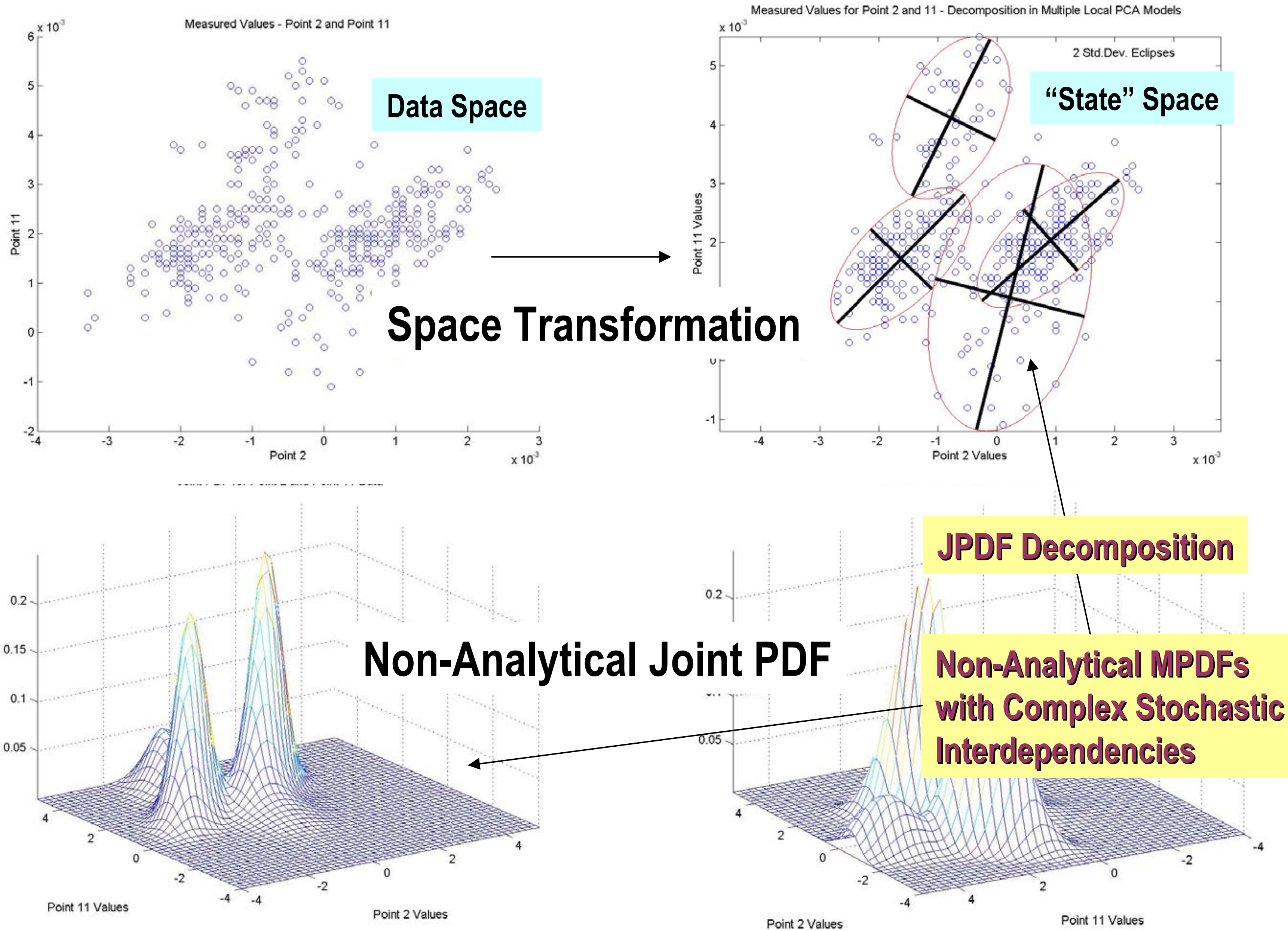
Decomposes overall complex JPDF in localized simple JPDFs.

Refined

Convergence: Using Maximumum Likelihood Function (in Probability sense)

Non-causal relationship

HO Stochastic Field Expansion in Local JPDF Models



Stochastic Physics-based ROM

The development of efficient Stochastic Physics-based ROMs includes:

1) Partitioning the original physical stochastic domain into subdomains (stochastic domain decomposition, substructuring)

2) Projecting the original stochastic solution onto reduced-size stochastic subspaces (stochastic projection, physics-based ROM)

Remark:
Physics-based stochastic ROM are extremely robust in comparison with perturbation methods (Taylor expansion, Neumann expansion, etc.) that are limited to local variations within the convergence radius of functions

Example:
CMU SNM Approach for mistuning uses a solution projection in an Eigen subspace.

2. Stochastic Perturbation Matrix Projection

Large Mistuning vs. Small Mistuning

The equations of motion in physical coordinates are

Stochastic Model

$$[\overline{\mathbf{K}} + \Delta\mathbf{K} + i\omega(\mathbf{C} + \mathbf{G}) - \omega^2(\overline{\mathbf{M}} + \Delta\mathbf{M}) + \mathbf{Z_a}]\boldsymbol{\alpha} = \mathbf{F}$$

Small (Frequency) Mistuning (standard ROM) assumes:

$$\Delta\mathbf{K} = \overline{\mathbf{K}}\,\varepsilon = \overline{\mathbf{K}}\,\varepsilon\,\mathbf{I} = \overline{\mathbf{K}}\boldsymbol{\varepsilon}$$ - *proportional variation* with $\underline{\mathbf{K}}$ RV

Notes: - *Matrix* ε *is diagonal* (same value for the blade/sector DOFs).

- Deviation ε is applied to E modulus (is the same for all DOFs)
- *"Local" blade modes maintain their shapes*

Large (Mode Shape) Mistuning (input calibration for ROM) assumes:

$$\Delta\mathbf{K} = \overline{\mathbf{K}}\boldsymbol{\varepsilon}$$ - *non-proportional variation* with **(** $\overline{\mathbf{K}}$ and $\overline{\mathbf{M}}$) SF

Remark:- *Matrix* $\boldsymbol{\varepsilon}$ *is non-diagonal.* It can be computed $\boldsymbol{\varepsilon} = \overline{\mathbf{K}}^{-1}\Delta\mathbf{K}$

- Deviation ε is applied to $\overline{\mathbf{K}}$ (and $\overline{\mathbf{M}}$) (is different for each DOF)
- "*Local" blade modes may change their shapes.*

Stochastic Perturbation Matrix Subspace Projection

Stochastic FEA Problem:

$$\mathbf{K}(\mathbf{x})\mathbf{u}(\mathbf{x}) = \mathbf{F}$$

For a random realization k of stochastic input vector $\mathbf{x}$ we can rewrite

$$[\overline{\mathbf{K}} + \Delta\mathbf{K}(\mathbf{x}_k)]\mathbf{u}(\mathbf{x}_k) = \mathbf{F}$$

The GPA subspace projection of the stochastic solution is:

$$\mathbf{u}(\mathbf{x}_k) = \sum_j y_j(\mathbf{x}_k)\,\boldsymbol{\varepsilon}(\mathbf{x}_k)^{j-1}\mathbf{u}(\overline{\mathbf{x}})$$

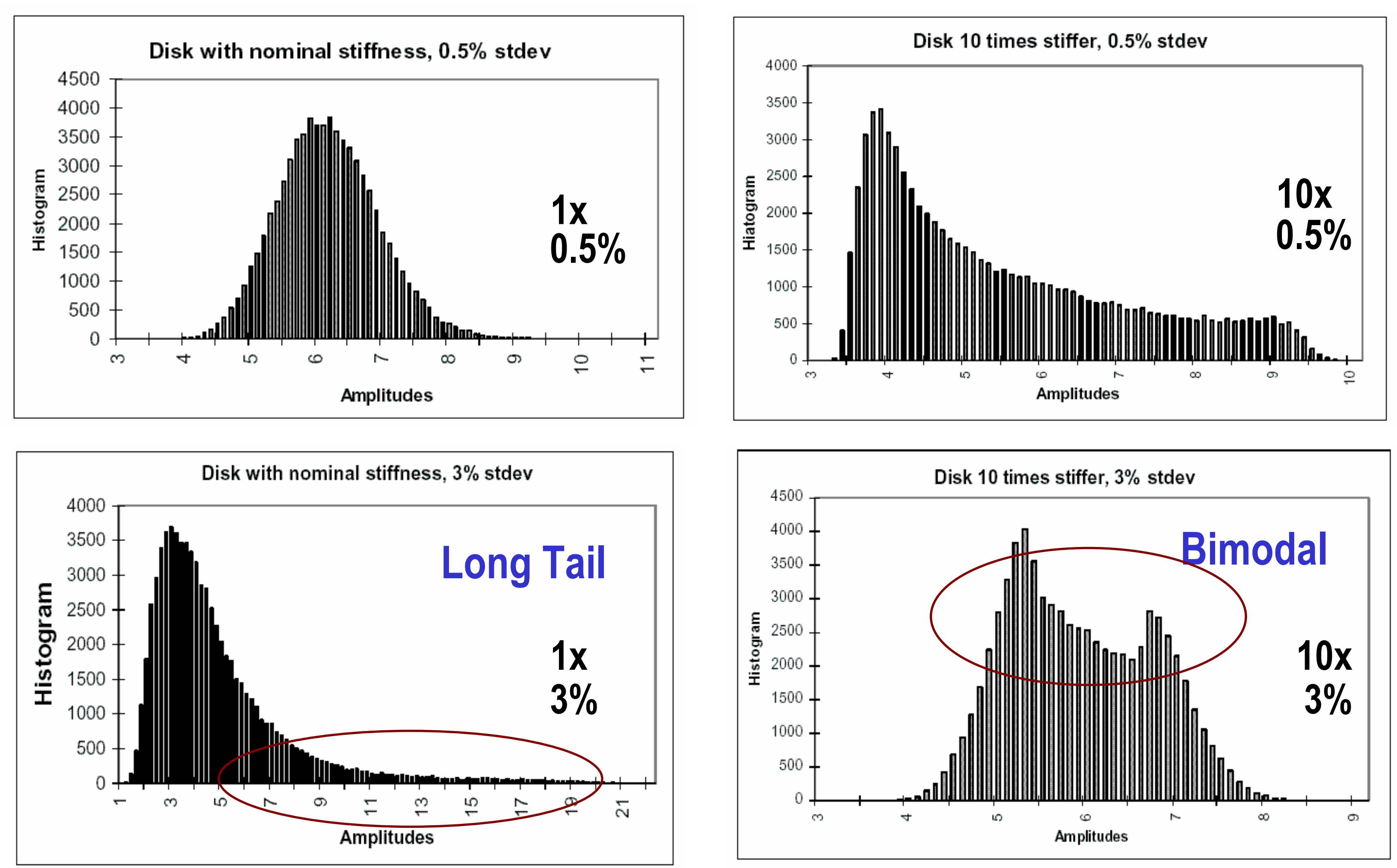
Few terms needed!

Note:

The expansion is fast-convergent if the probability densities of random eigen values of the matrix $\overline{\mathbf{K}}^{-1}[\overline{\mathbf{K}} + \Delta\mathbf{K}(\mathbf{x}_k)]$ are highly overlapping; only few terms are needed, or equivalently the GPM ROM size is very reduced.

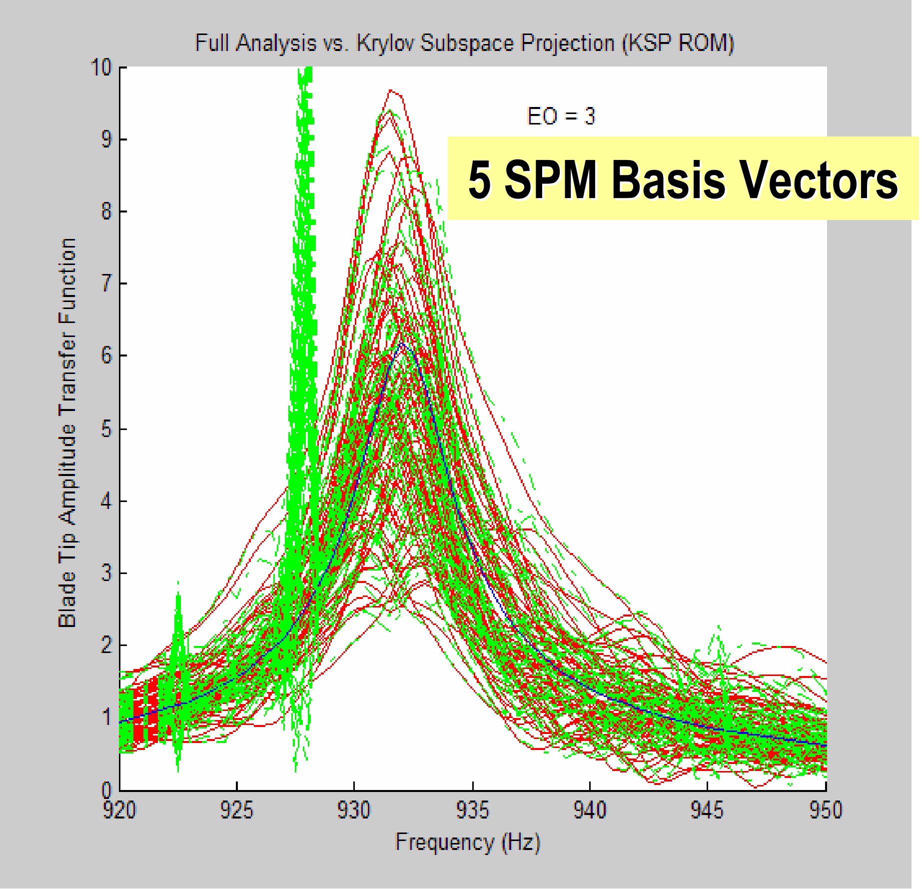
3. The 72 Blade Compressor Blisk Example

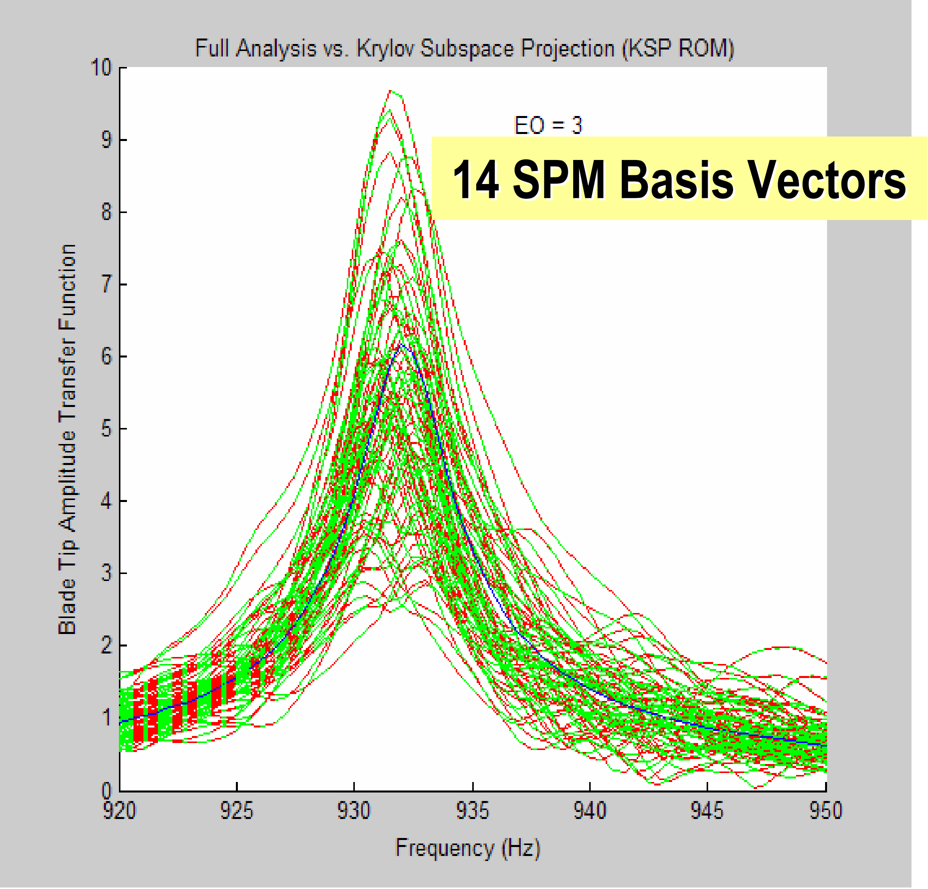
PDF of Mistuned Rotor Blade Tip Amplitude Responses



Stochastic Perturbation Matrix ROM Solution for A 72 Blade Blisk System

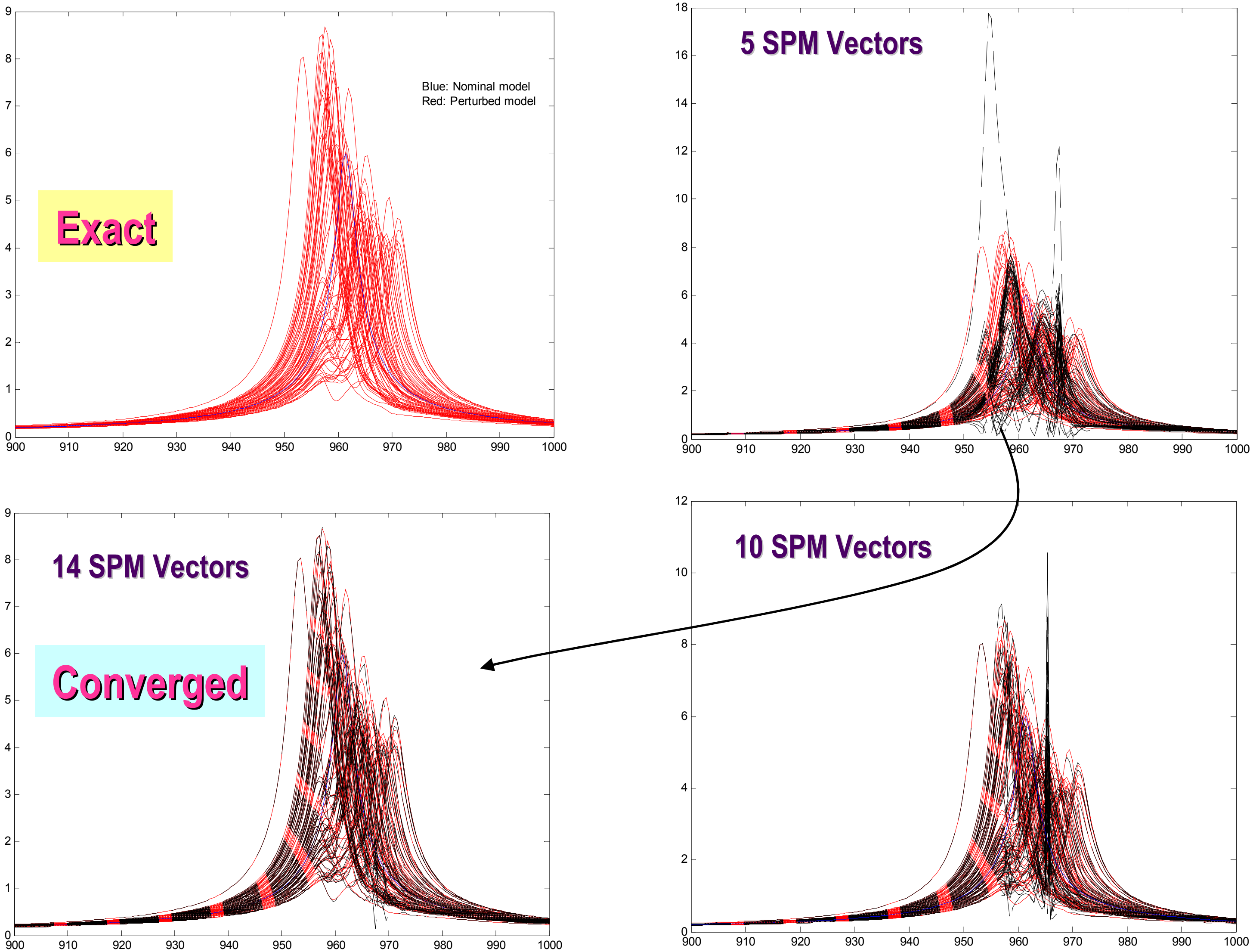
LO Bending Blade Mode Family Mistuning





SPM ROM Solution for Modified Blisk System

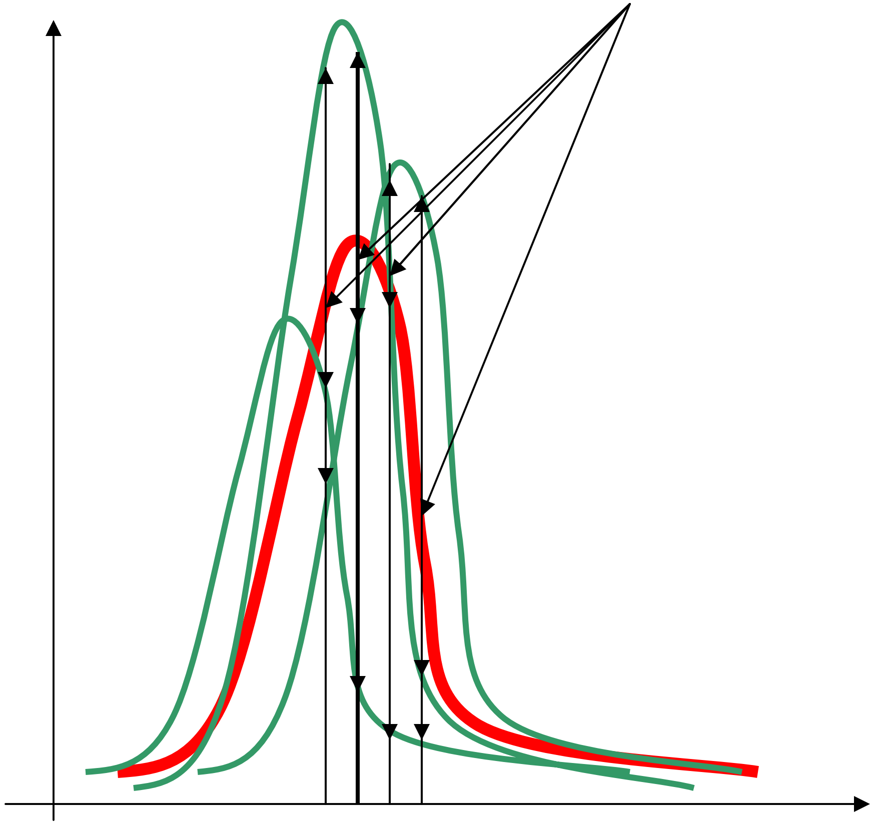
LO Bending Blade Mode Mistuning for Modified Disk (Stiffer)

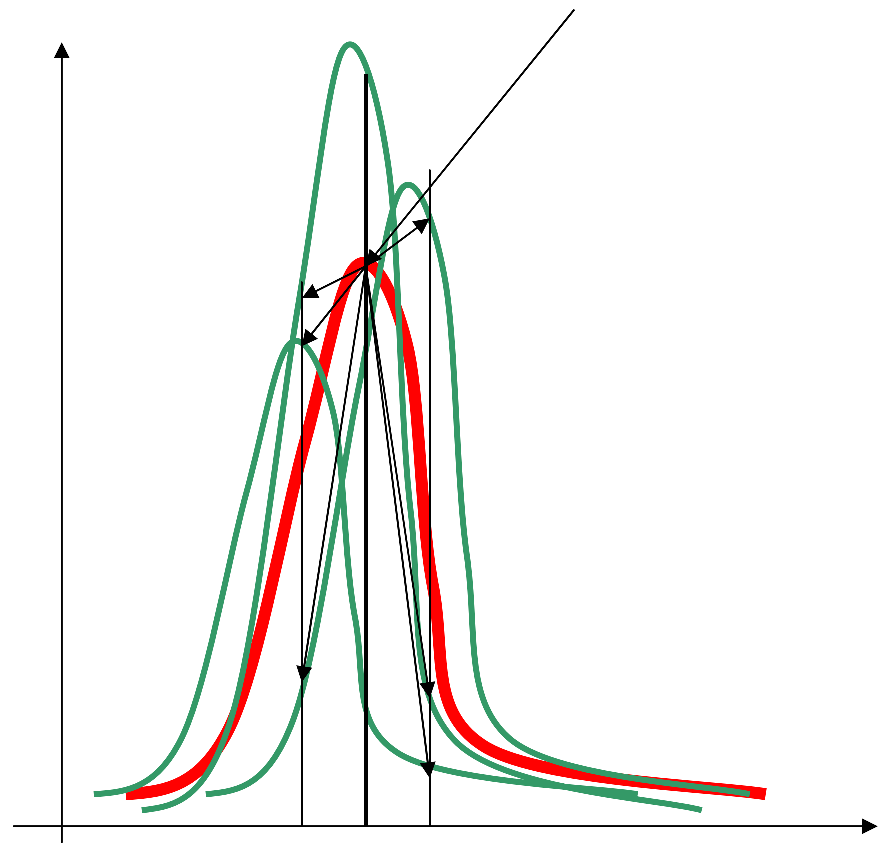


SPM ROM Solution Robustness Study

Multiple Starting Frequency Points

Single Starting Frequency Point

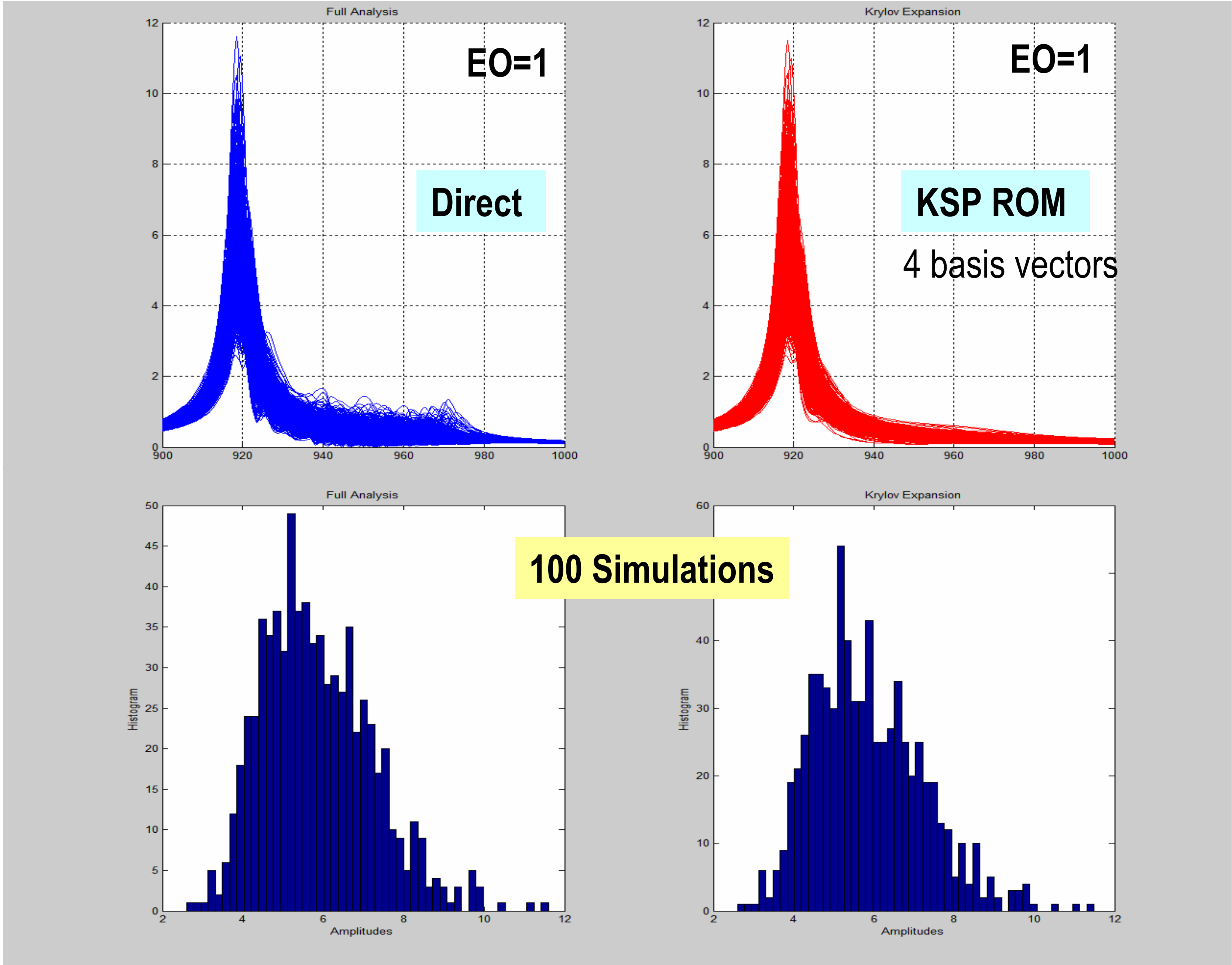




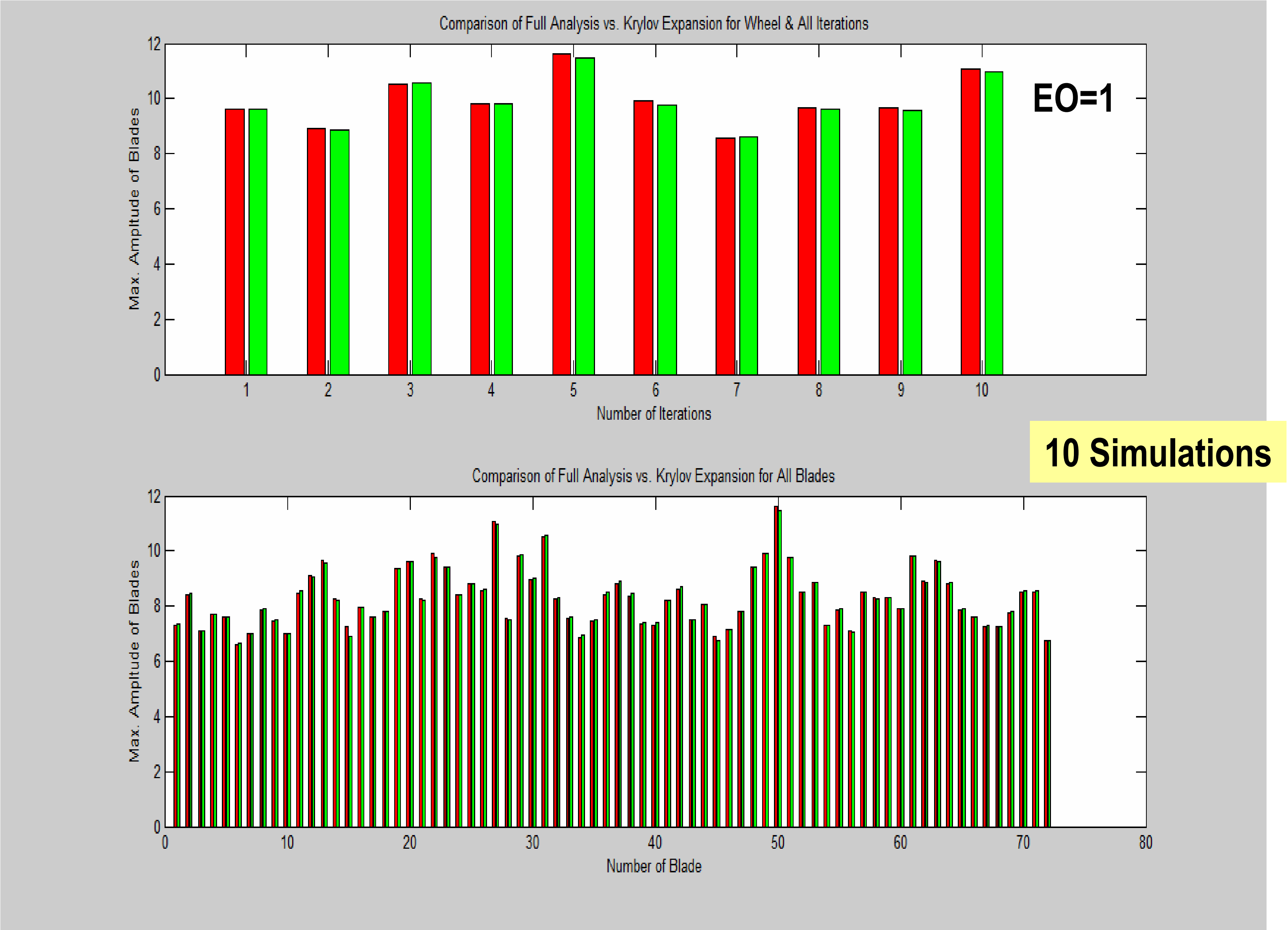
$$K(\omega, \Delta t) = \overline{K}(\omega) + \Delta K(\omega, \Delta t)$$

$$K(\omega^* + \Delta\omega, \Delta t) = \overline{K}(\omega^*) + \Delta K(\omega^* + \Delta\omega, \Delta t)$$

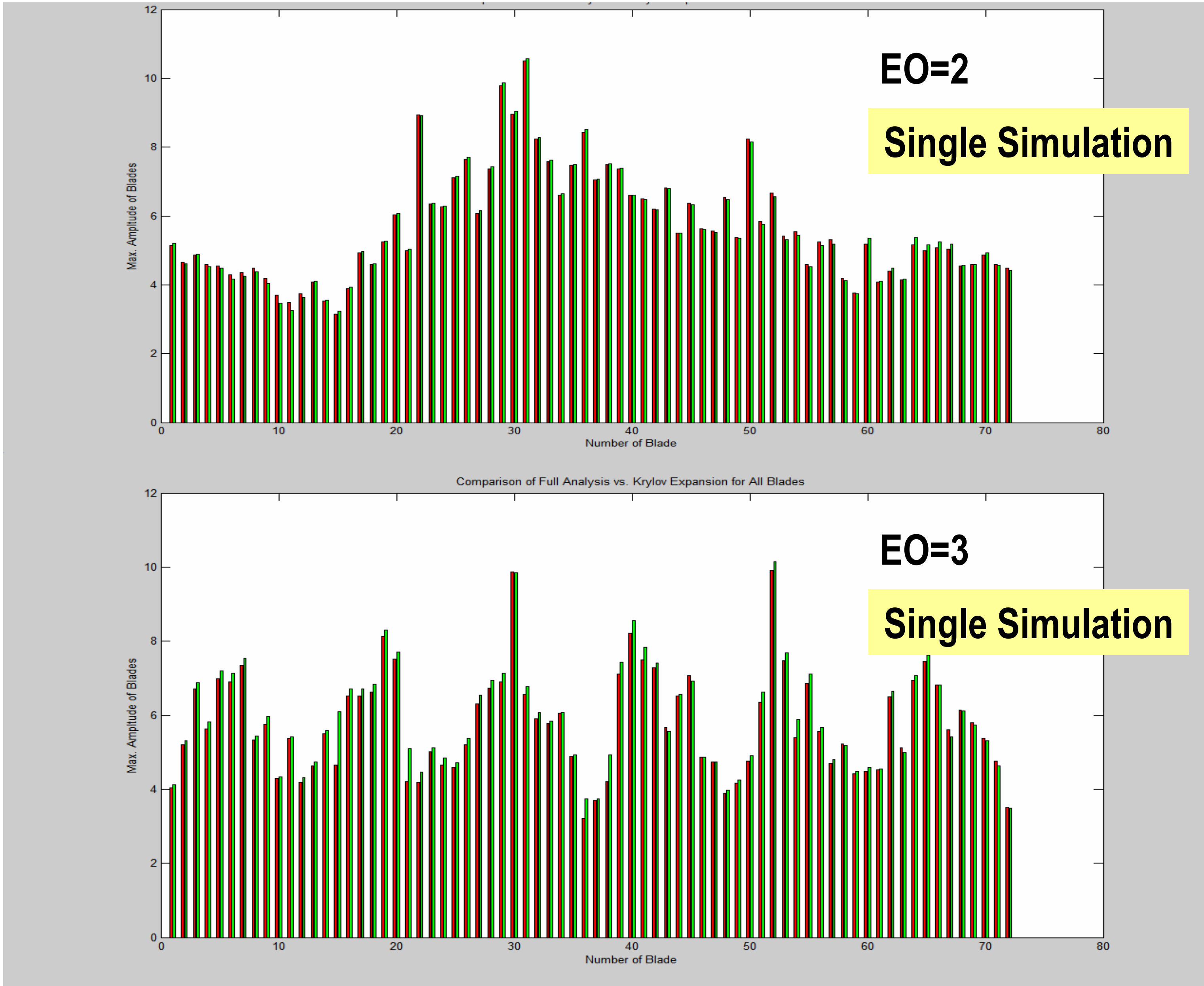
Single Frequency Point SPM ROM Solution



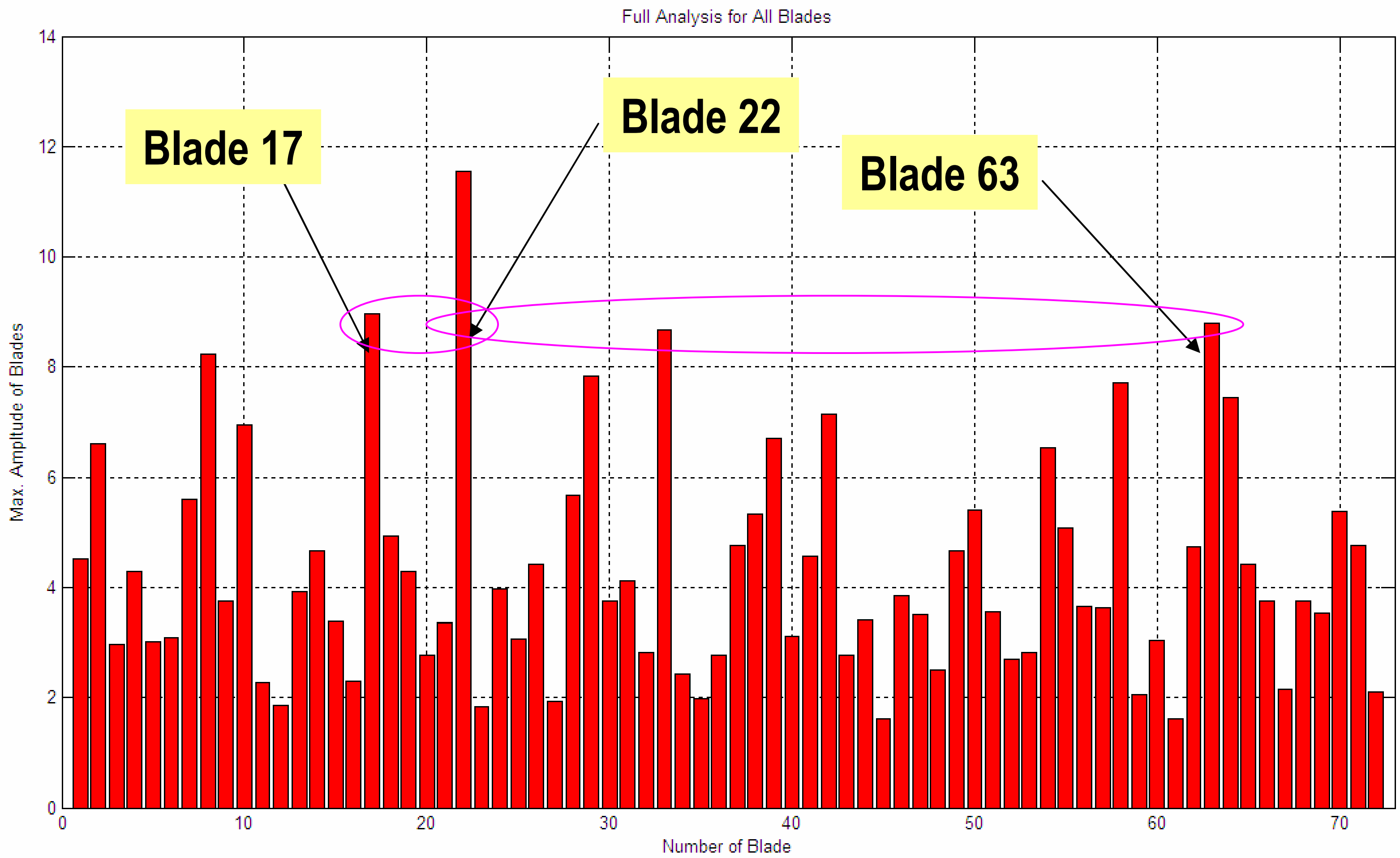
Single Frequency Point SPM ROM Solution



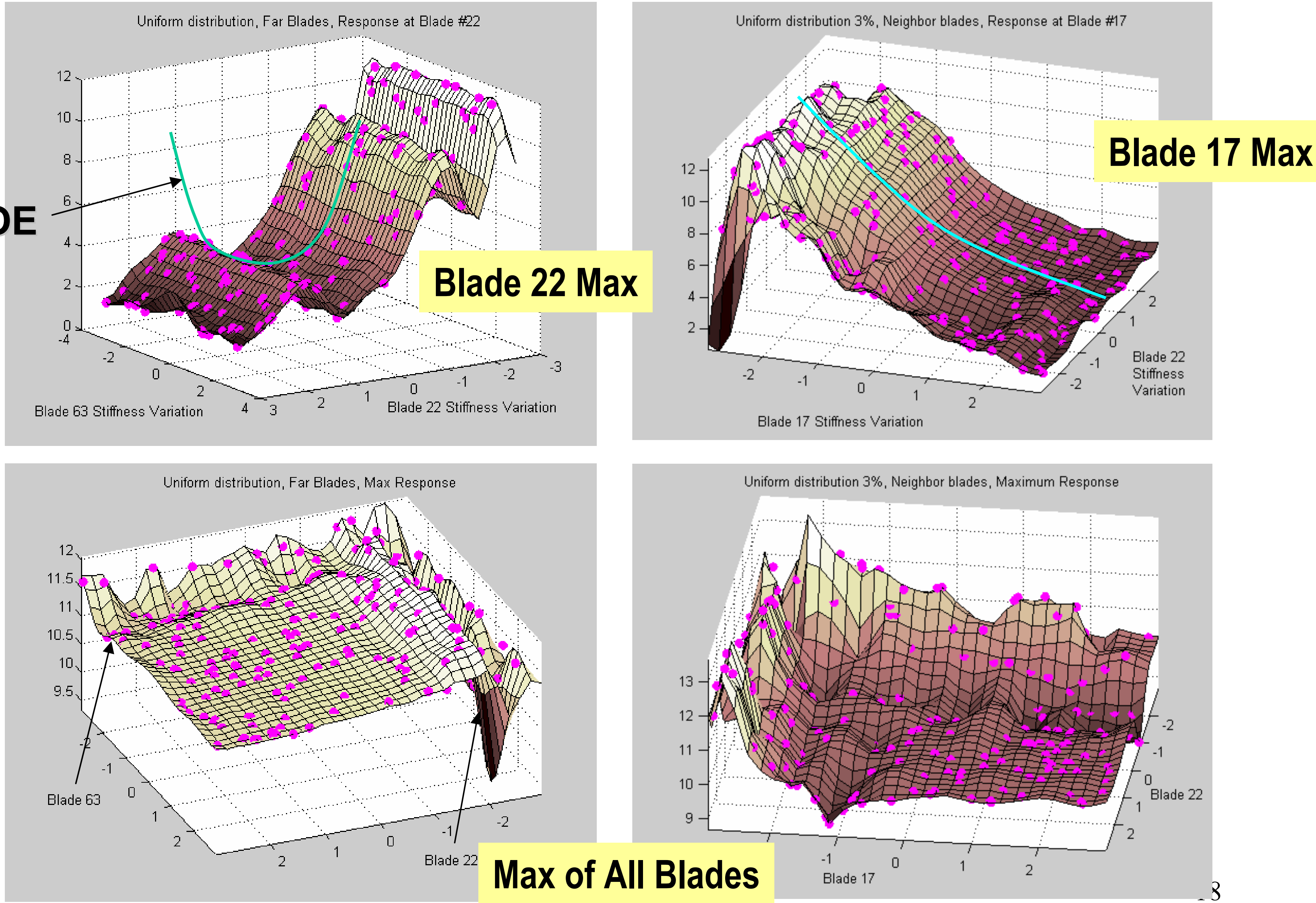
Single Frequency Point SPM Solution



Application of SPM ROM Approach to Maintenance of Geometrically Mistuned IBRs



SPM ROM for Studying Blade Geometry Variation Effects



4. Concluding Remarks

1. Stochastic projection schemes represent systematic mathematical procedures for building powerful physics-based stochastic ROMs that are highly applicable to bladed-disk mistuning problems.

2. For large mistuning problems, the proposed Stochastic Perturbation Matrix (SPM) approach is an extremely accurate and fast prediction tool. SPM ROM has an extremely fast convergence, since the size of the required ROM is very reduced. For the illustrated (mean-based PC) 72 blade blisk system case study, the typical SPM ROM size for computing accurate results was from 5 to 20 equations.

3. The SPM ROM approach is perfectly fitted for solving large mistuning problems, for both low-order and high-order system modes, including complex dynamic couplings in veering regions. The author believes that the SPM ROM approach will play a gradually increasing role in future for solving difficult, large mistuning problems.