



## SIMPLIFIED MODELING OF EFFECTS OF CONCRETE CRACKING ON OUT-OF-PLANE VIBRATIONS OF FLOORS

Luben Todorovski<sup>1</sup>, Mahmoud Khoncarly<sup>2</sup>, Dan M Ghiocel<sup>3</sup>, and Michael Donohoe<sup>4</sup>

<sup>1</sup> Consulting Engineer, URS Corporation, Princeton, NJ (lubentodorovski@urs.com)

<sup>2</sup> Manager of Discipline Engineering, URS Corporation, Cleveland, OH

<sup>3</sup> Chief of Engineering, Ghiocel Predictive Technologies, Rochester, NY

<sup>4</sup> Engineer, URS Corporation, Princeton, NJ

### ABSTRACT

A simplified approach is presented for addressing effects of concrete cracking on the out-of-plane vibrations of floors in frequency domain Soil-Structure Interaction (SSI) analyses of reinforced concrete shear wall structures. As input for the design of equipment and components supported by flexible slabs, vertical In-Structure Response Spectra (ISRS) are developed by enveloping nodal responses from shell elements representing full stiffness properties of uncracked concrete slabs, and the response of added Single Degree of Freedom (SDOF) oscillators representing the out-of plane response of the slabs with reduced bending stiffness due to concrete cracking. Responses obtained from SSI analyses of Finite Element (FE) models with full slab stiffness and SDOF oscillators are compared to those of FE models with reduced out-of-plane slab stiffness to assess the effectiveness of the SDOF modeling approach to capture effects of slab cracking. The comparisons indicate the SDOF modeling approach can be an effective tool for capturing peak frequency shifts in the design ISRS that are due to slab cracking.

### INTRODUCTION

Under static design loads and low to moderate intensity seismic inputs, the out-of-plane bending stiffness of the floors and walls can be reduced due to concrete cracking that can result in spectral peak frequency shifts of the ISRS used for design of equipment and components supported by flexible slabs. Large non-linear FE models using complex constitutive models to represent the non-linear concrete behavior are required to accurately capture the effects of concrete cracking on the seismic SSI response of nuclear structures. The extensive computational effort and uncertainties related to the modeling of the non-linear constitutive behavior, coupled with appropriate consideration of loading conditions with time, make these analyses not suitable for practical design applications.

In lieu of performing explicit non-linear SSI analyses, an alternative approach is implemented for development of vertical ISRS for design of equipment and components supported by flexible slabs that include the effects of concrete cracking. In order to include in the design ISRS the shifts of spectral peak frequencies that are due to the cracking of the concrete, the responses obtained from slab shell elements with uncracked concrete properties are enveloped with the response of an added SDOF oscillator representing the response of cracked slab with bending stiffness reduced by 50% as stipulated by ASCE/SEI 43-05 code.

### MODELING APPROACH

SDOF oscillators are introduced in a FE model of a typical Reactor Building (R/B) structure with uncracked concrete stiffness properties to address the effects of possible concrete cracking on the ISRS used for design of equipment and components supported by flexible slabs. The R/B is a reinforced concrete shear wall type of structure resting on a reinforced concrete basemat that is shared with the containment structures. The FE model of the R/B structure consists of shell elements representing the slabs and the walls and beam elements representing the reinforced concrete and steel columns and beams.

A separate FE model of the R/B is developed with identical FE mesh where the properties of slab shell elements are adjusted to account for the reduction of the out of plane stiffness while maintaining a full (uncracked concrete) stiffness of the slabs in-plane direction. The following relationships are used to adjust the properties of the FE of the slab with isotropic linear elastic properties so that the in-plane axial stiffness of the slab to be  $n_a$  times the initial stiffness ( $n_a \cdot t_m \cdot E_m \propto t_0 \cdot E_0$ ) and the out-of-plane flexural stiffness be  $n_b$  times the initial stiffness ( $n_b \cdot t_m^3 \cdot E_m \propto t_0^3 \cdot E_0$ ):

$$E_m = E_0 \cdot \sqrt{\frac{n_b}{n_a^3}}; \quad t_m = t_0 \cdot \sqrt{\frac{n_a}{n_b}}; \quad \gamma_m = \gamma_0 \cdot \sqrt{\frac{n_b}{n_a}} \quad (1)$$

where:  $E_m$ ,  $t_m$  and  $\gamma_m$  are the adjusted Young's modulus, thickness and, unit weight properties of the shell and  $E_0$ ,  $t_0$  and  $\gamma_0$  are the corresponding initial properties of the shell FE.  $n_b = 2$  and  $n_a = 1$  are substituted in Eq. 1 to obtain the following properties of cracked slab with 50% reduced out-of-plane stiffness while the in-plane stiffness remains unchanged:

$$E_m = E_0 \cdot \sqrt{2}; \quad t_m = t_0 / \sqrt{2}; \quad \gamma_m = \gamma_0 \cdot \sqrt{2} \quad (2)$$

The dynamic properties of the SDOF oscillators are obtained from results of modal analyses of floor models extracted from the model with reduced out-of plane stiffness of slabs. Each one of the R/B major floor elevations is isolated, and boundary conditions are established as shown in Figure 1. The upper and lower boundaries of the floor models are restrained in the horizontal direction to accurately mimic the bending stiffness at the wall/slab interfaces. The horizontal and vertical displacements at the junctions of the slab with the supporting walls are also restrained to eliminate the effects of the axial stiffness of the walls on the modal analyses results and to disregard the slab horizontal modes of vibrations. Where the slab is supported by columns, the vertical displacement is constrained. Modal analyses of the isolated floor models provide natural frequencies for the vertical vibrations of the floor slabs.

The slabs for which the first dominant mode frequency ( $f_{cracked}$ ) is less than 50 Hz in the cracked concrete condition are considered flexible per Section 3.1.1 of DC/COL-ISG-01. A SDOF oscillator is developed for each of the flexible slabs supporting equipment or components. SDOF oscillators are modeled independent of slabs and consist of a unit lumped mass supported by a number of springs with stiffness in the global vertical direction as shown in Figure 2. The very small unit SDOF mass ensures that the dynamic properties of the model remain unchanged. These SDOFs are incorporated in the R/B FE model with full stiffness properties by connecting the SDOF springs to nodes located at the intersection of slabs with walls.

Since unit mass is assigned to the SDOF oscillators, the stiffness of the vertical springs is computed as follows:

$$k_{cracked} = \frac{4}{n_s} \cdot f_{cracked}^2 \cdot \pi^2 \quad (3)$$

where  $n_s$  is the number of SDOF springs that connect the lumped mass to the FE model.

## EVALUATION OF SDOF MODELING APPROACH

In order to validate the accuracy of the SDOF models and evaluate the effectiveness of the SDOF modeling approach for representing out-of-plane responses of cracked slabs, SSI analyses are performed on the model with full stiffness properties and SDOF oscillators and the model with reduced stiffness properties. Both models are resting on the surface of a very stiff elastic half-space simulating fixed base conditions. Acceleration transfer functions and 5% damping ISRS are calculated for the responses of the

SDOF masses in the full stiffness model and for the response of the cracked slabs FE in the model with reduced stiffness at nodal locations where the slabs experience the largest out-of-plane vibrations. To evaluate the effectiveness of SDOF approach for different types of design ground motions, ISRS are produced using acceleration time histories compatible to the modified RG 1.60 and high frequency design ground motion response spectra (DGMRS). Figure 3 presents the horizontal and vertical components of the modified RG 1.60 and high frequency DGMRS normalized by the peak ground accelerations (PGA).

The peak frequencies of the acceleration transfer functions obtained from analyses of SDOF models are checked first to ensure the accuracy of the stiffness properties assigned to the SDOF models. The comparison of transfer function and ISRS results reveals the ability of the SDOF to capture responses of cracked slabs. Figure 4, 5 and 6 present the comparisons of transfer function and ISRS results for three representative slab locations for the two types of input ground motion considered. ISRS are normalized with respect to the Zero Period Acceleration (ZPA) value of the ISRS calculated for the FE node of the model with reduced stiffness.

A set of SSI analyses are performed on the model with full stiffness properties and SDOF oscillators for the three profiles representing best estimate (BE), lower bound (LB) and upper bound (UB) rock properties of a typical nuclear power plant site. From the results of these analyses, ISRS are developed for design of equipment and components supported by flexible slabs as the envelope of vertical responses of the nodes located at all slab corners, the mid-span node where the slabs experience the largest out-of-plane vibrations, and the SDOF mass node.

In order to further evaluate how effective the SDOF approach is in capturing the concrete cracking effects, the design ISRS are compared with ISRS obtained from the SSI analyses of the model with reduced slab out-of-plane stiffness for the BE, LB and UB profiles. The ISRS representative of out-of-plane response of cracked slabs are developed as the envelope of the vertical response of shell nodes located at the slab corners and the mid-span node. Before being compared, the ISRS obtained from the SSI analyses of BE, LB and UB profiles are enveloped. Responses obtained from the three components of the earthquake are combined using the square root of the sum of squares method. Figures 7 and 8 present the comparisons of the ISRS for the three typical slabs obtained from SSI analyses of BE, LB and UB rock profiles for modified RG 1.60 DGMRS and high frequency DGMRS, respectively. The ZPA value of the design ISRS are used to normalize the cracked slabs ISRS.

## CONCLUSIONS

The comparisons indicate that the SDOF modeling approach can be an effective tool for capturing peak frequency shifts in the design ISRS that are due to slab cracking. The SDOF can underestimate the amplitude of the resonant responses of mid-span FE nodes where the slab experiences largest vibrations. ISRS obtained using the SDOF approach are still adequate for design of equipment and components that do not have a point support at the middle of the slab. It should be noted that for sites with high frequency ground motion, higher modes of vibration can be significant, resulting in high frequency spectral peaks that cannot be captured by the SDOF that represent only the first mode of vibration of the slab. As shown in Figure 8, these higher modes of vibration of the cracked slab can be addressed by enveloping the responses of uncracked and cracked slab for different soil conditions. Additional investigation is needed to further evaluate and improve the SDOF approach for sites with high frequency DGMRS.

## REFERENCES

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- US Nuclear Regulatory Commission Regulatory Guide RG 1.60. (1973). *Design Response Spectra for Seismic Design of Nuclear Power Plants*.

US Nuclear Regulatory Commission DC/COL-ISG-01 (2008) *Interim Staff Guidance on Seismic Issues Associated with High Frequency Ground Motion in Design Certification and Combined License Applications.*

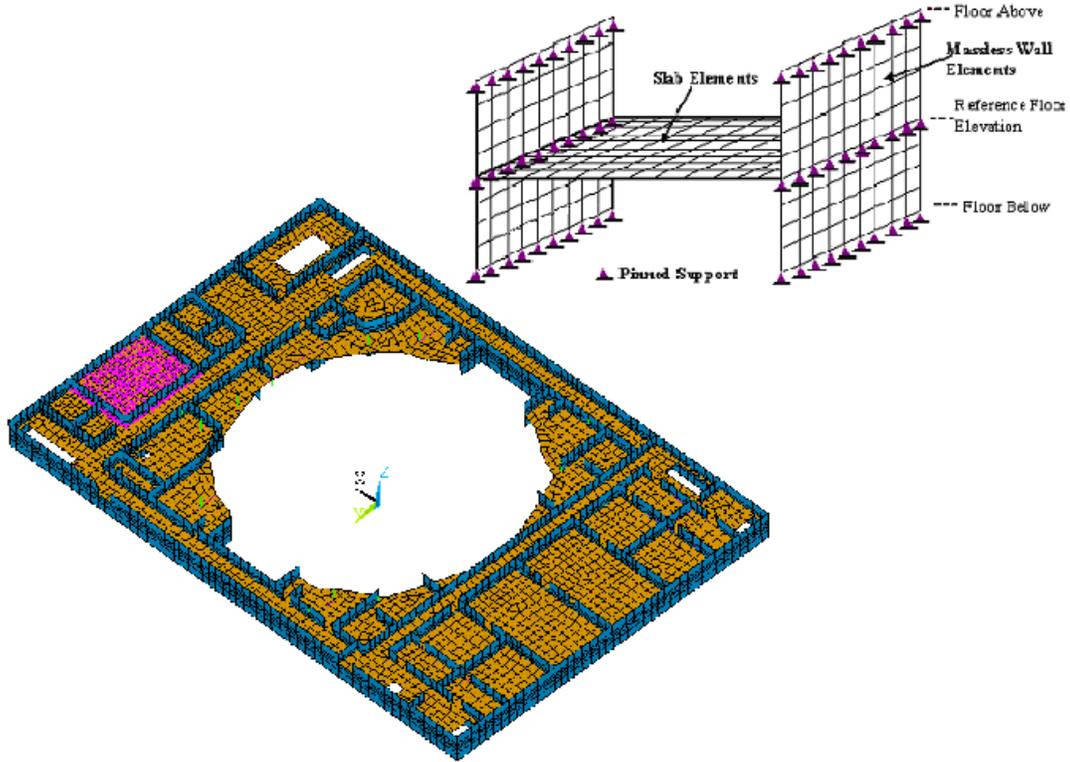


Figure 1. Extracted FE Model of Floor Slabs with Boundary Conditions

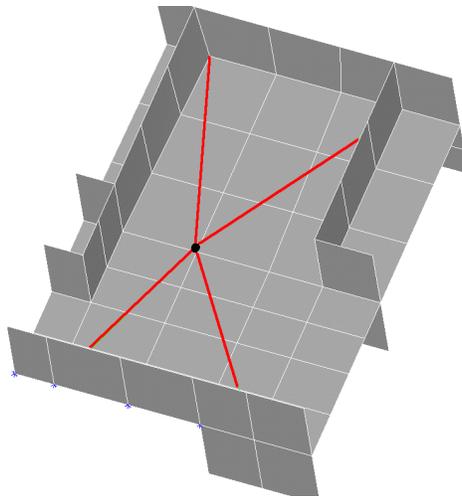


Figure 2. SDOF Oscillator in FE Model with Uncracked Concrete Properties

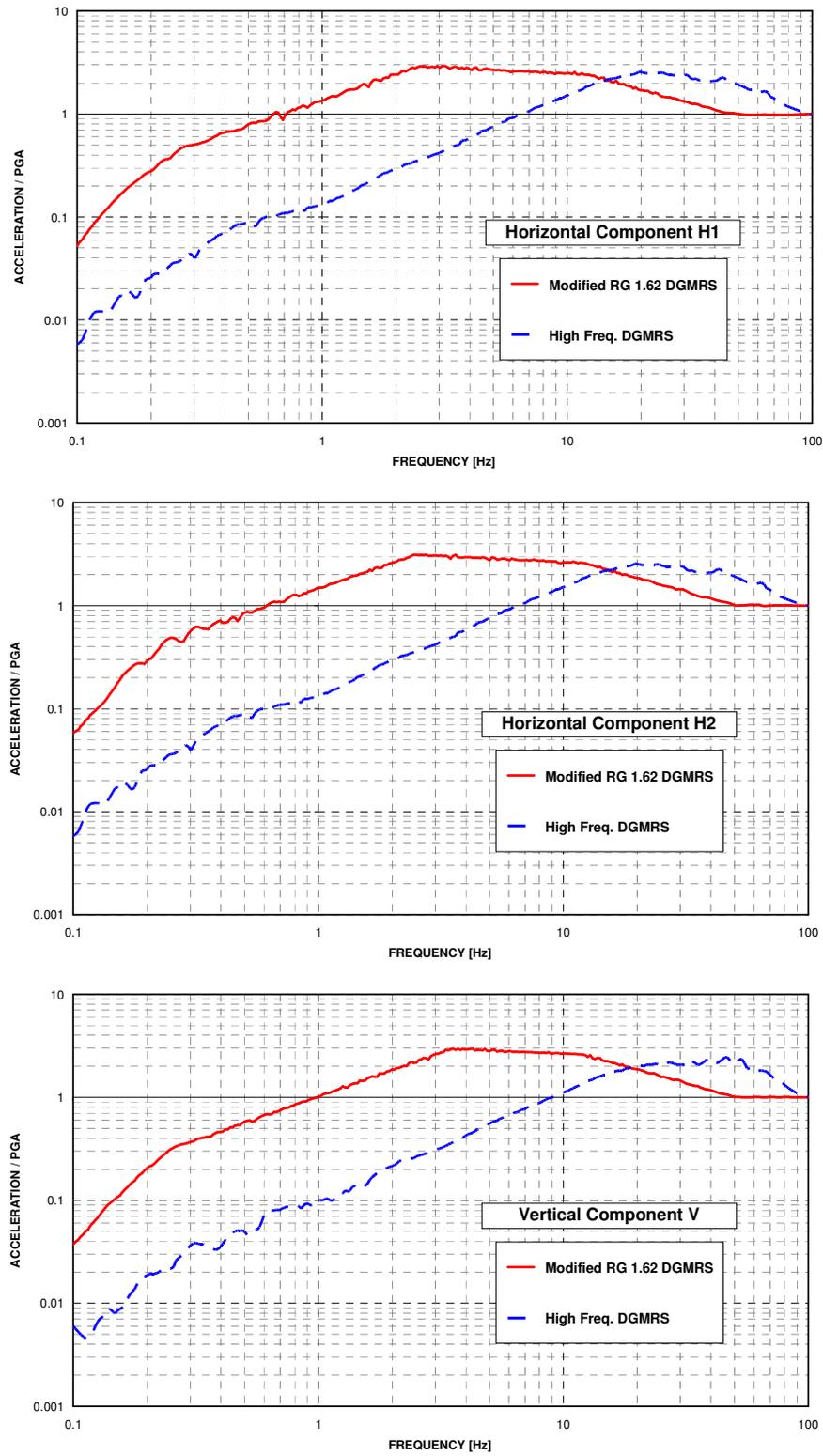


Figure 3. Input Design Ground Motion Response Spectra (DGMRS)

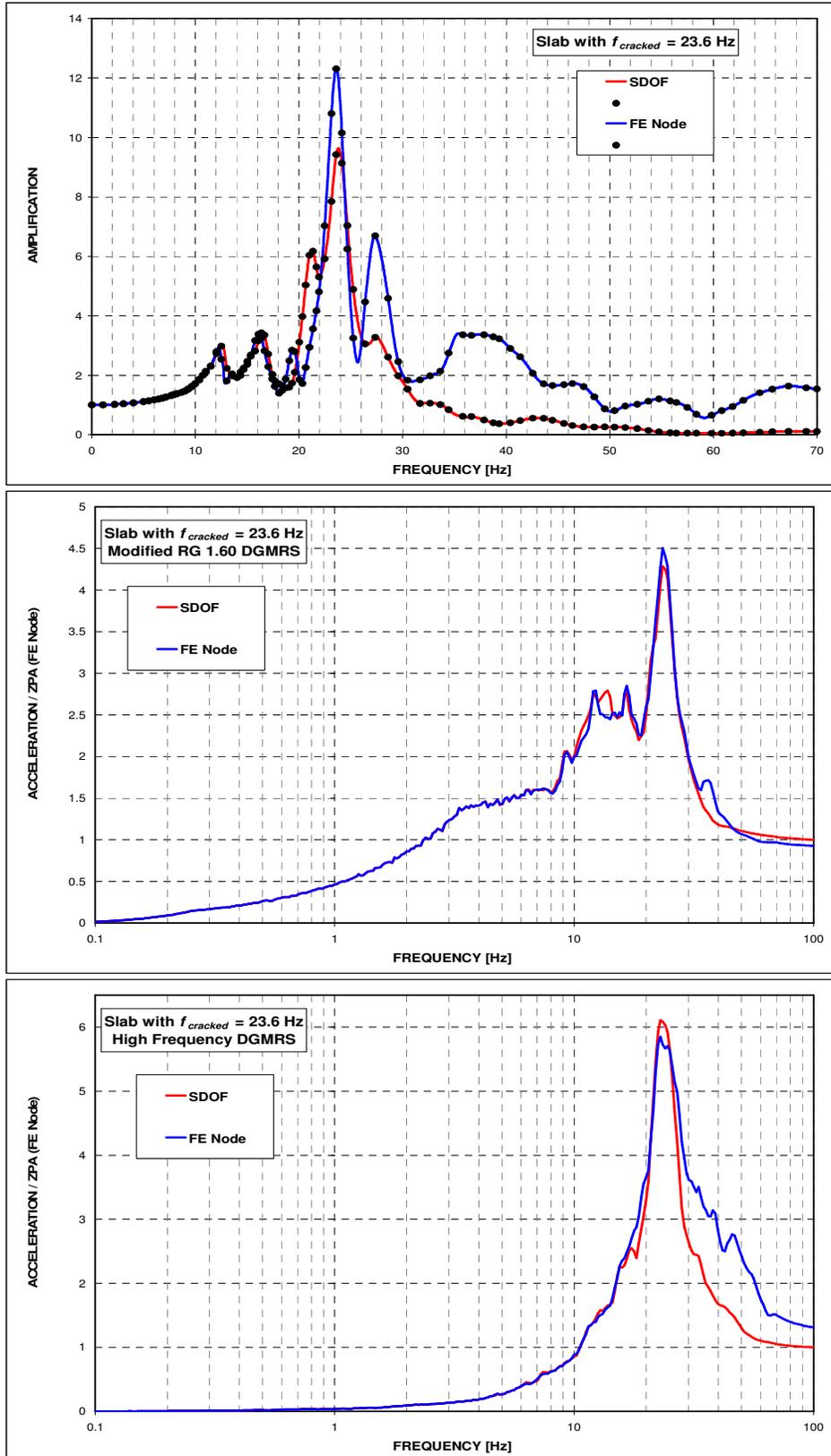


Figure 4. Fixed Base Responses Slab with  $f_{cracked} = 23.6$  Hz

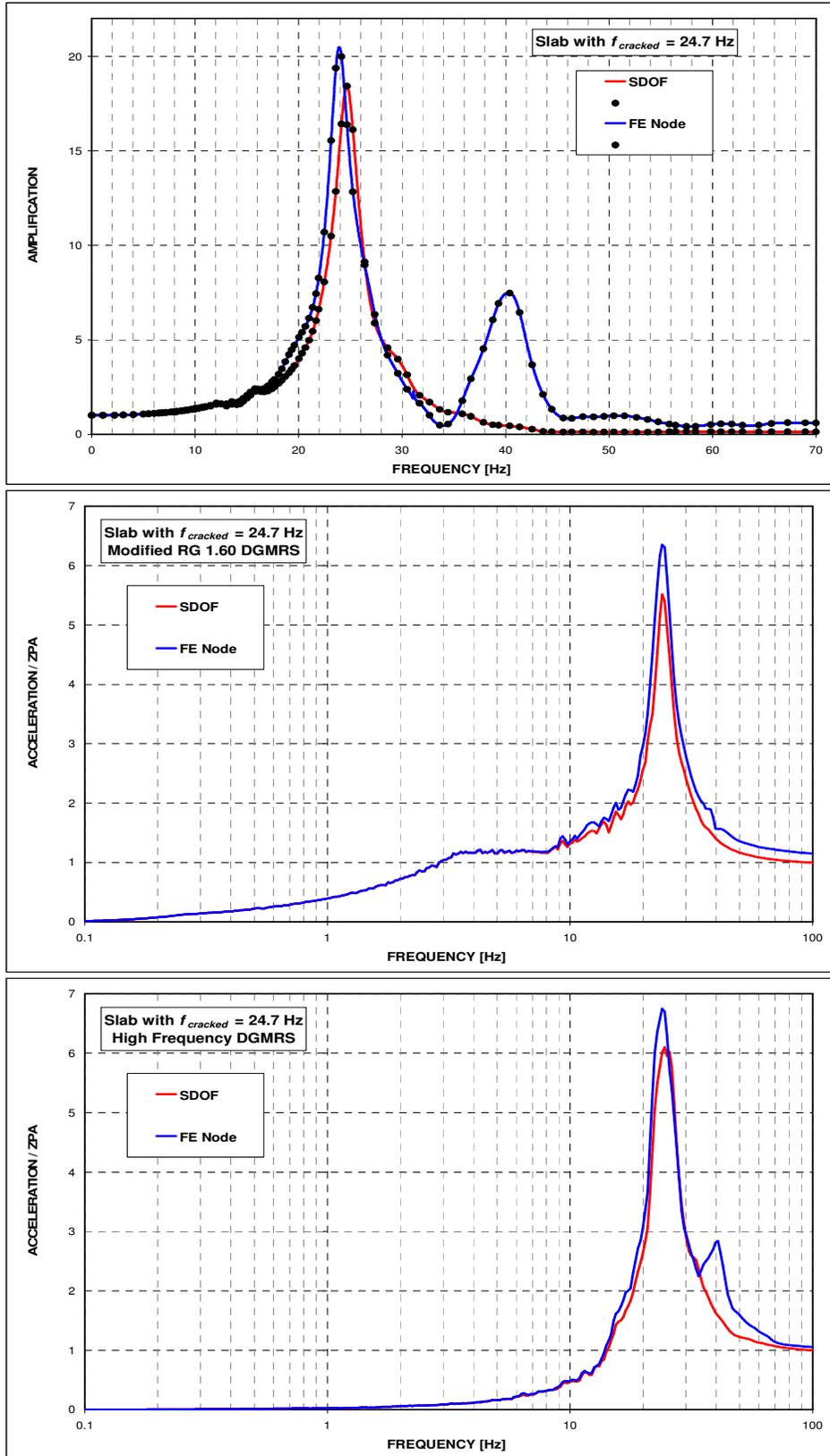


Figure 5. Fixed Base Responses Slab with  $f_{cracked} = 24.7$  Hz

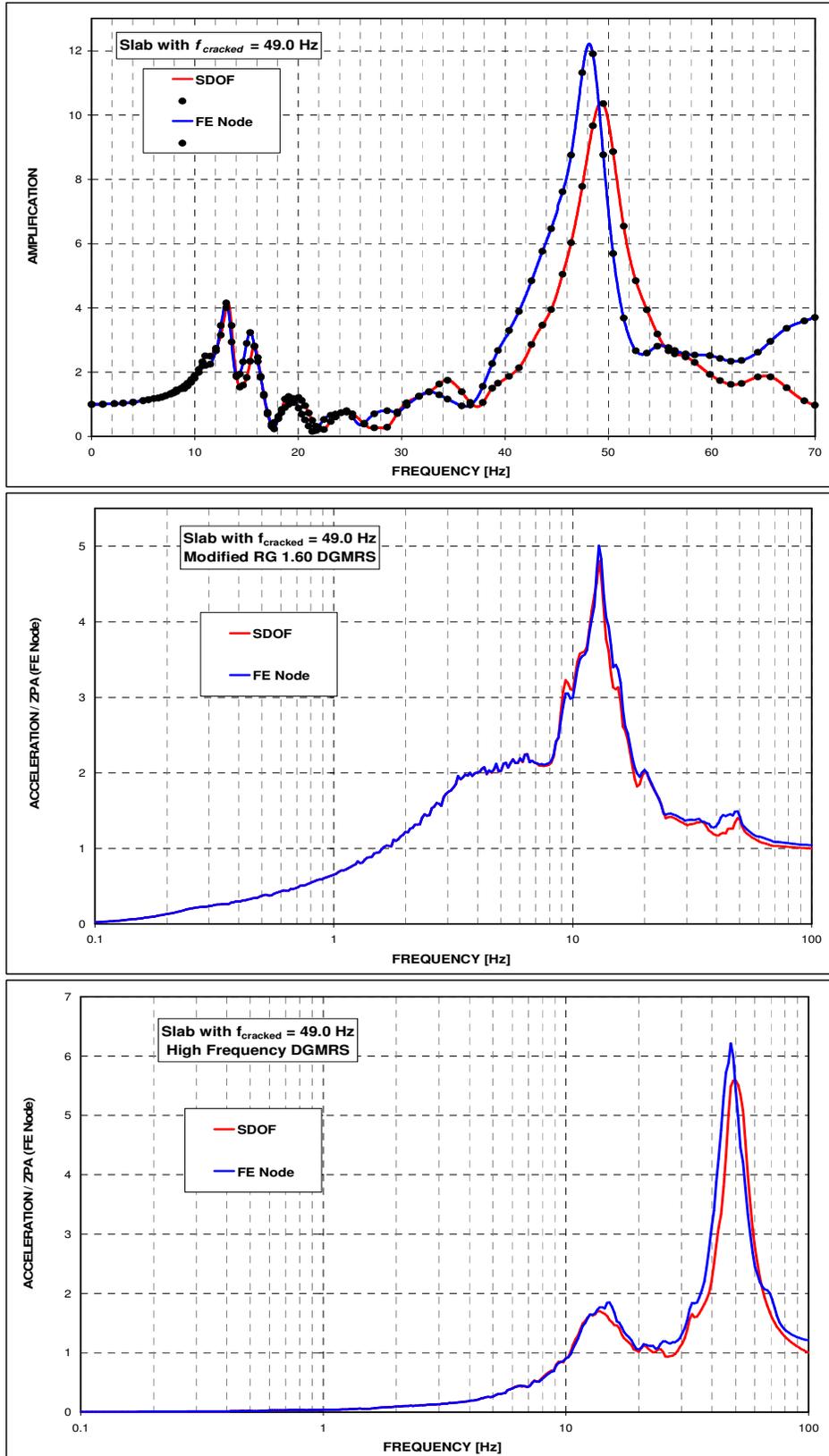


Figure 6. Fixed Base Responses Slab with  $f_{cracked} = 49.0$  Hz

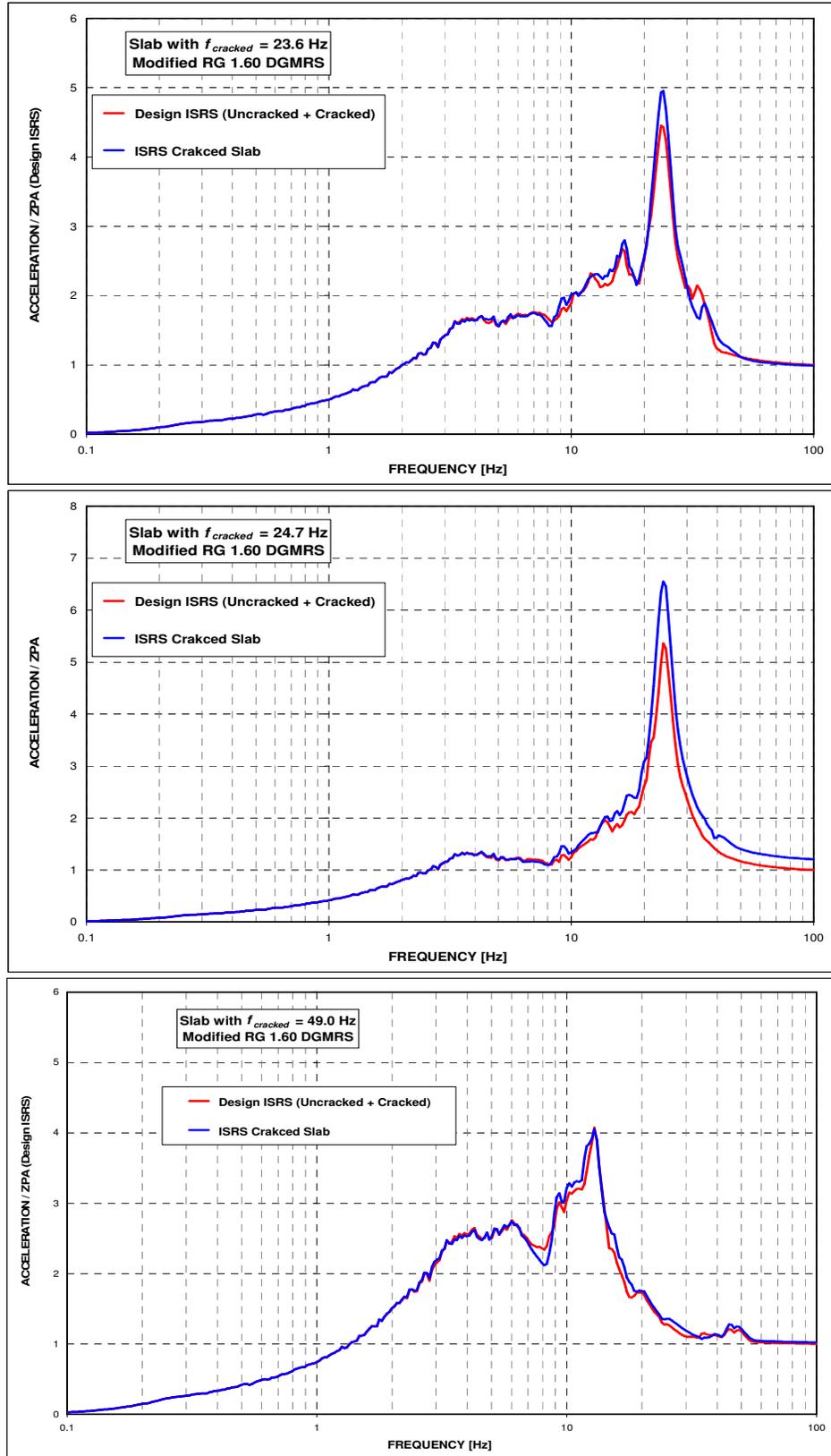


Figure 7. Comparisons ISRS from Modified RG 1.60 DGMRS

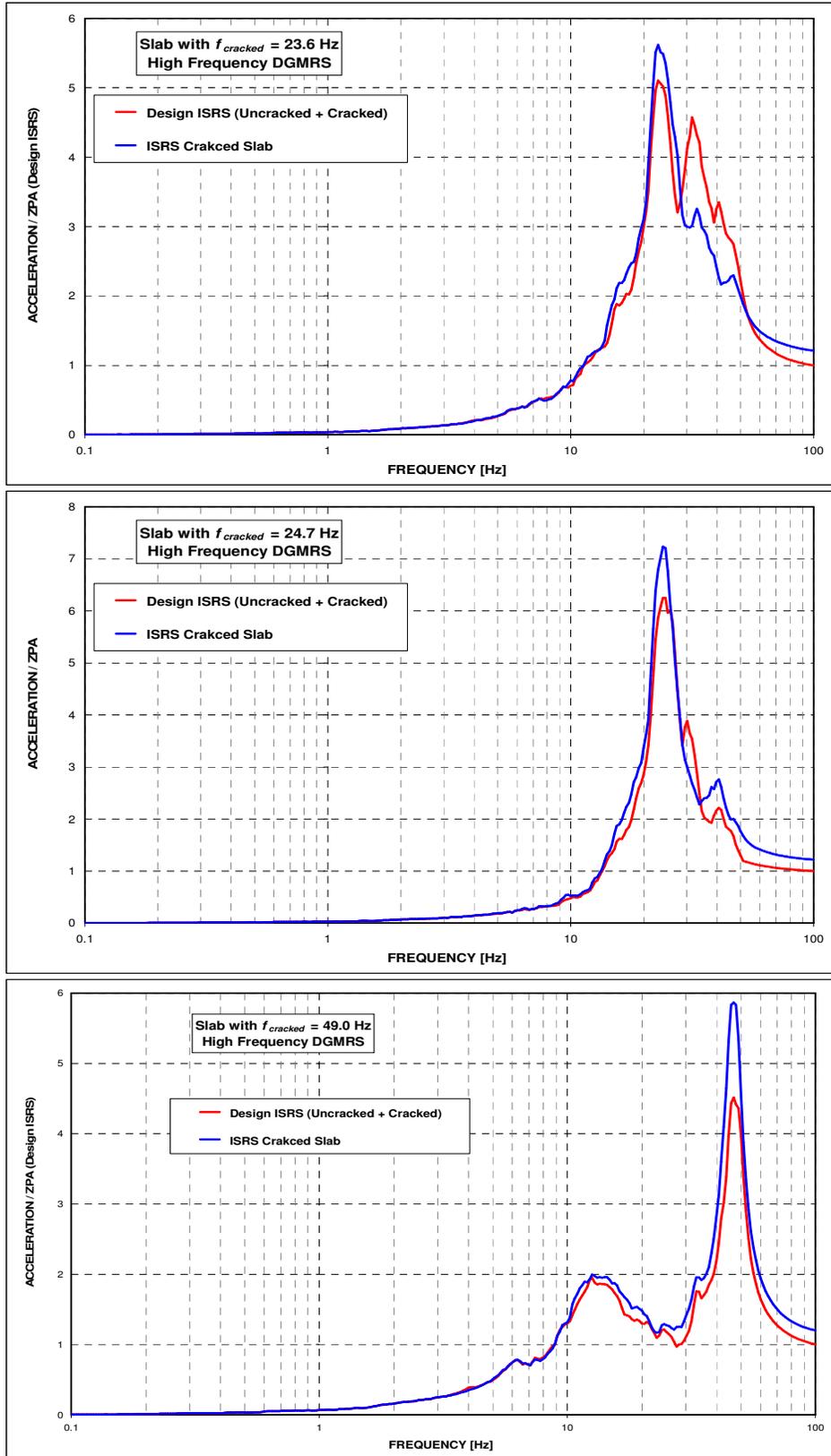


Figure 8. Comparisons ISRS from High Frequency DGMRS