Stochastic Response Surface Approximation Using (Bayesian and Fuzzy) Hierarchical Models

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Objective of this Presentation

To discuss some of advanced stochastic response approximation tools that can useful for engine applications.

The proposed hierarchical models are capable of accurately approximating complex stochastic response problems.

The use of hierarchical approximation models is very efficient when employed in conjunction with (i) stochastic ROM and for (ii) RBDO analysis.

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Content Presentation

1. Basic Computational Stochastic Mechanics Issues

2. Stochastic Response Approximation
   - Two-Level Hierarchical Stochastic Models
   - Three-Level Hierarchical Stochastic Models

3. Application Examples

4. Concluding Remarks

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1. Basic Stochastic Mechanics Issues

Two Major Stochastic Modeling Aspects:

1. Develop ***Accurate Stochastic Approximation Models*** for High-Complexity Behavior Given Sample Dataset (Data-*based Stochastic ROM*)
   - Global and Local Accuracy in Statistical Data Space – *ROM in Data Space*
   - For both *System Inputs* and Outputs

2. Develop ***Fast Stochastic Simulation Models*** Given the Physics (PDE) and Stochastic Inputs (*Physics-based Stochastic ROM*)
   - Global and Local Accuracy in Physical Space – *ROM in Physical Space*
   - For *System Outputs*

3. Combine 1) and 2) to build efficient Stochastic ROMs for stress predictions

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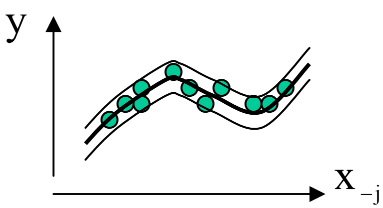
2. Stochastic Response Surface Approximation

Second-Order (SO) Approximation of Stochastic Fields

Explicit Formulation: Using function approximation via nonlinear regression



$$\boxed{y|\mathbf{x} = r(\mathbf{x}) + \varepsilon(\mathbf{x})}$$

--- Least-square fitting (y is explicit)

Limited

$$E[y|\mathbf{x}] = r(\mathbf{x})$$

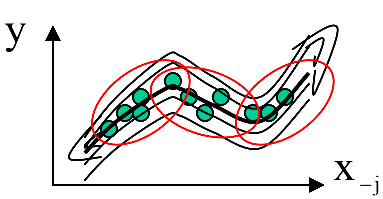
Defined by stochastic vector or field

Convergence: Minimizing Mean Square Error (in Mean Square sense)

Causal relationship. Implicit assumption of Gaussian variations.

High-Order (HO) Approximation of Stochastic Fields

Implicit Formulation: Using joint PDF estimation of $z = [y, \mathbf{x}]^T$



$$\boxed{f(y|\mathbf{x}) = \frac{f(y, \mathbf{x})}{f(\mathbf{x})} =}$$

--- Stochastic Neural Networks (y is implicit)

Decomposes overall complex JPDF in localized simple JPDFs.
Solution is obtained by stochastic interpolation

Refined

$$E[y|\mathbf{x}] = \int_{-\infty}^{\infty} y f(y|\mathbf{x}) dy$$

Convergence: Using Maximumum Likelihood Function (in Probability sense)

Non causal relationship. No implicit assumption of Gaussian variations.

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SO Stochastic Field Expansion

A Non-Gaussian (translation) stochastic field can be expanded:

$$u(\mathbf{x},\theta) = \sum_i u_i(\mathbf{x}) \underset{\text{Non-Gaussian}}{z_i(\theta)} = \underset{\substack{\text{Static}\\\text{Mapping}}}{g}\Big(\sum_i u_i(\mathbf{x}) \underset{\text{Gaussian}}{z_i(\theta)}\Big)$$

Case (1) **Case (2)**

Example: KL/PCA Expansion

$$u(\mathbf{x},\theta) = \sum_{i=0}^{n} \sqrt{\lambda_i}\,\Phi_i(\mathbf{x}) z_i(\theta) = \sum_{i=0}^{n} u_i(\mathbf{x}) z_i(\theta)$$

KL/PCA
Eigen Decomposition
REDUCED STOCHASTIC SPACE

(1) Original Space Expansion

Compute Non-Gaussian Variables: $z_i(\theta) = \int_D u_i(x)\, u(x,\theta)\, dx$

(2) Transformed Space Expansion

a. Transform Original Field in A Gaussian Image

b. Perform Expansion in Gaussian Image Space

c. Back-Transform to Non-Gaussian Original Space

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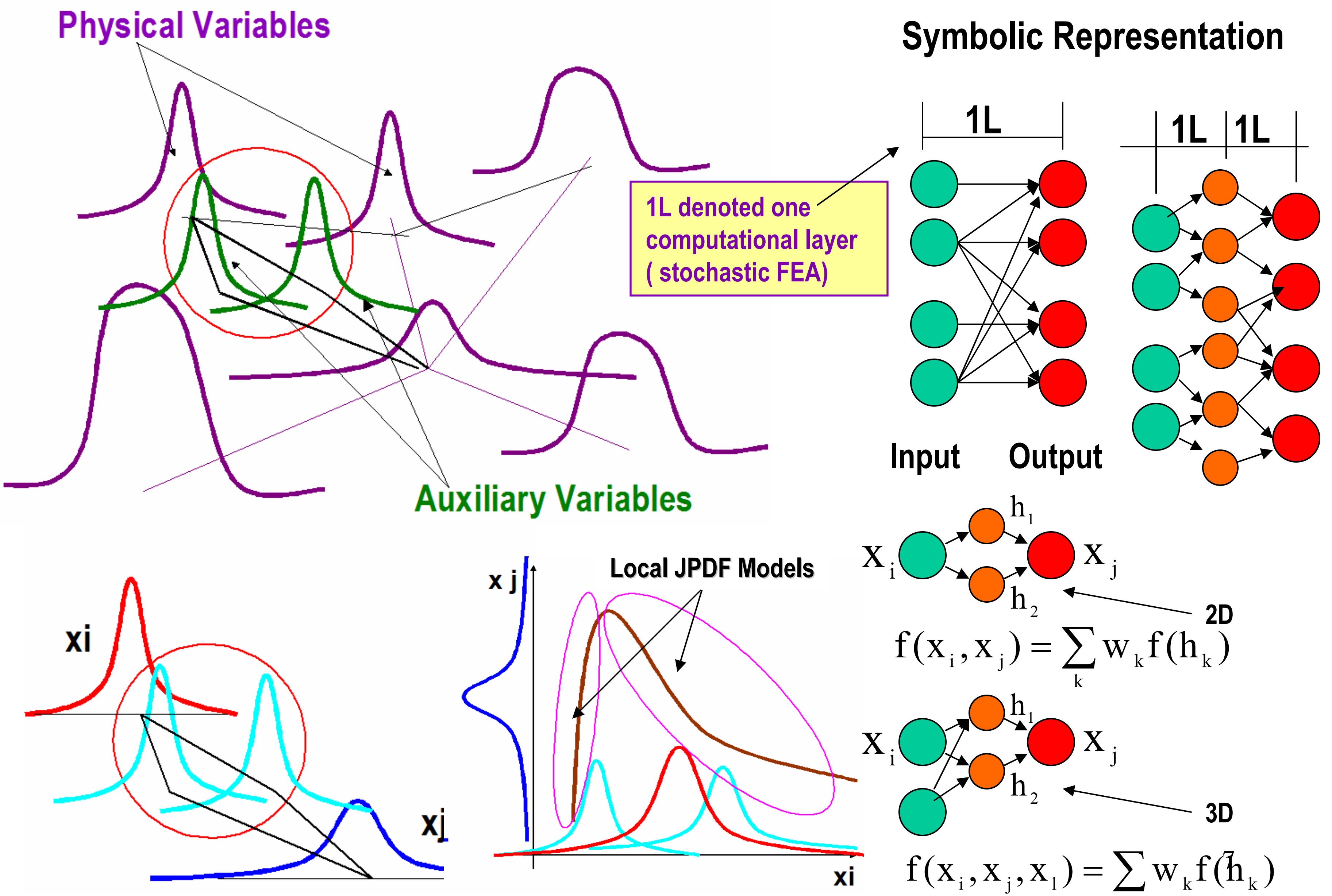
Two-Level (Bayesian) Hierarchical Models



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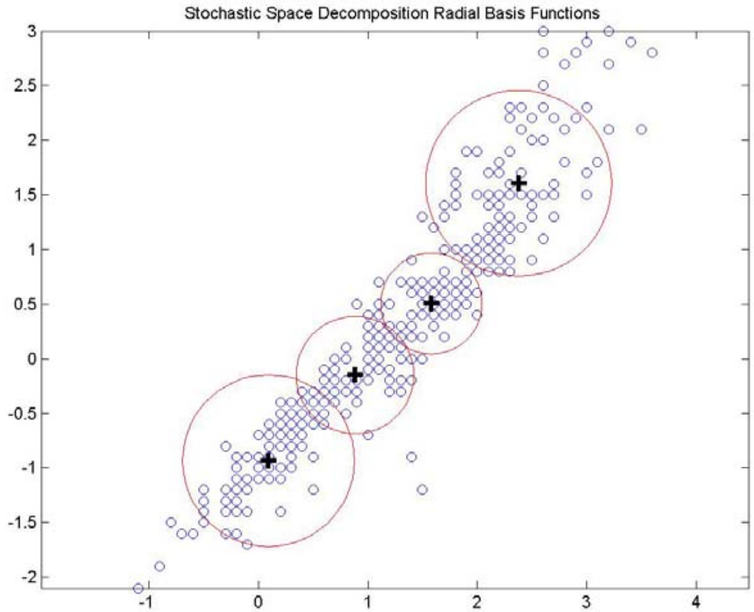
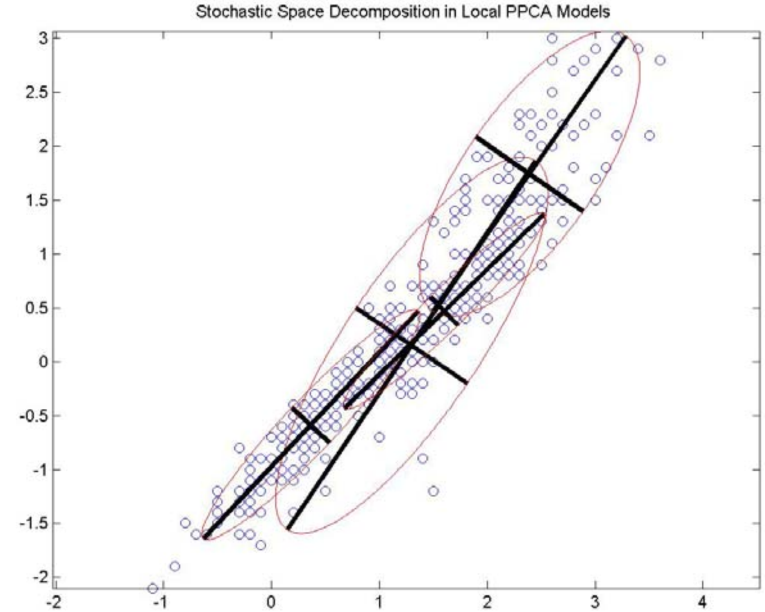
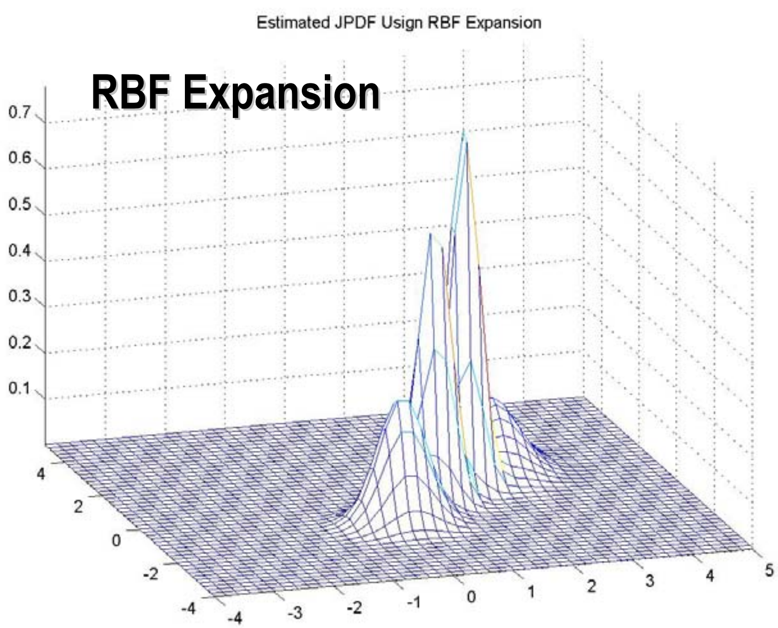
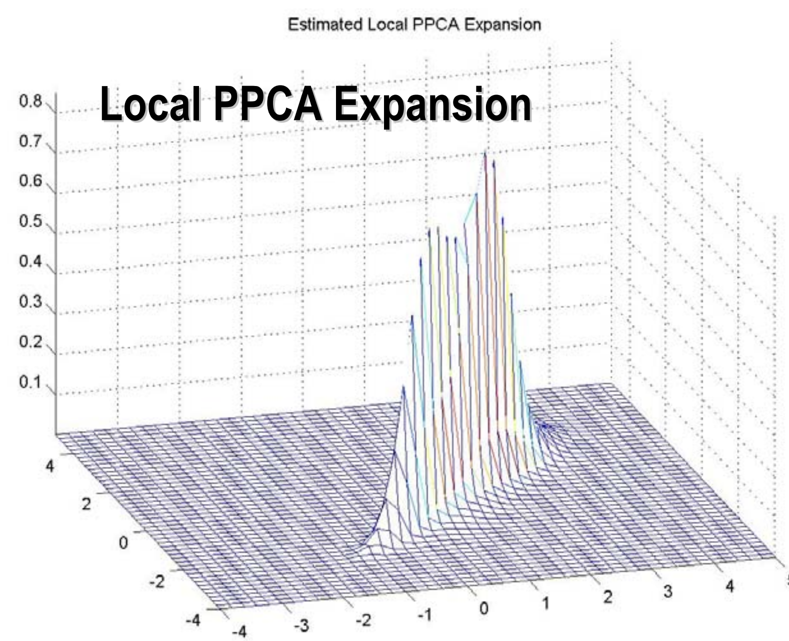
Comparison between RBF and Local PPCA Expansions



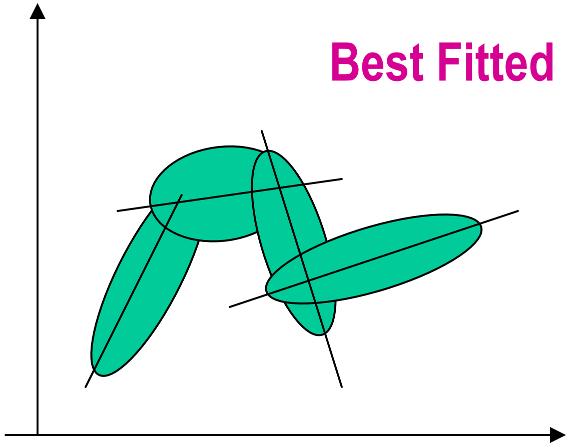


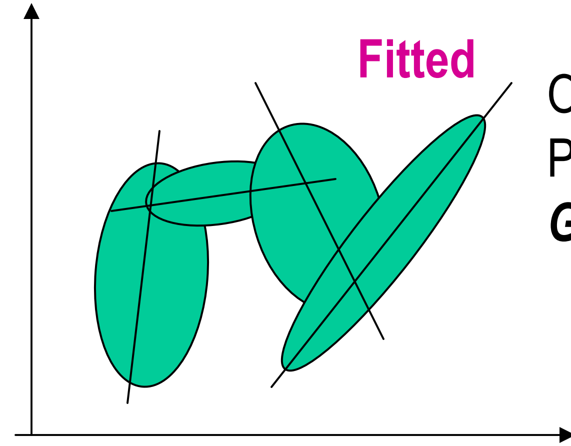




Stochastic Model Fitting and Selection

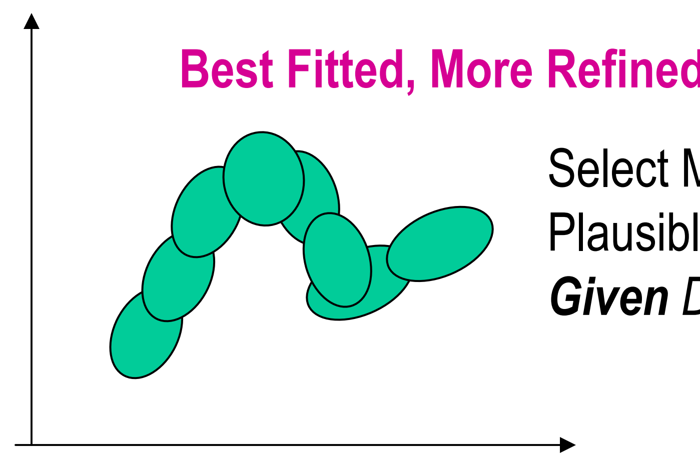
Model Fitting (Estimation Problem)





Optimize Model Parameters ***Given*** *Data*

Model Selection (Evidence Problem)



Select Most Plausible Model ***Given*** *Data*

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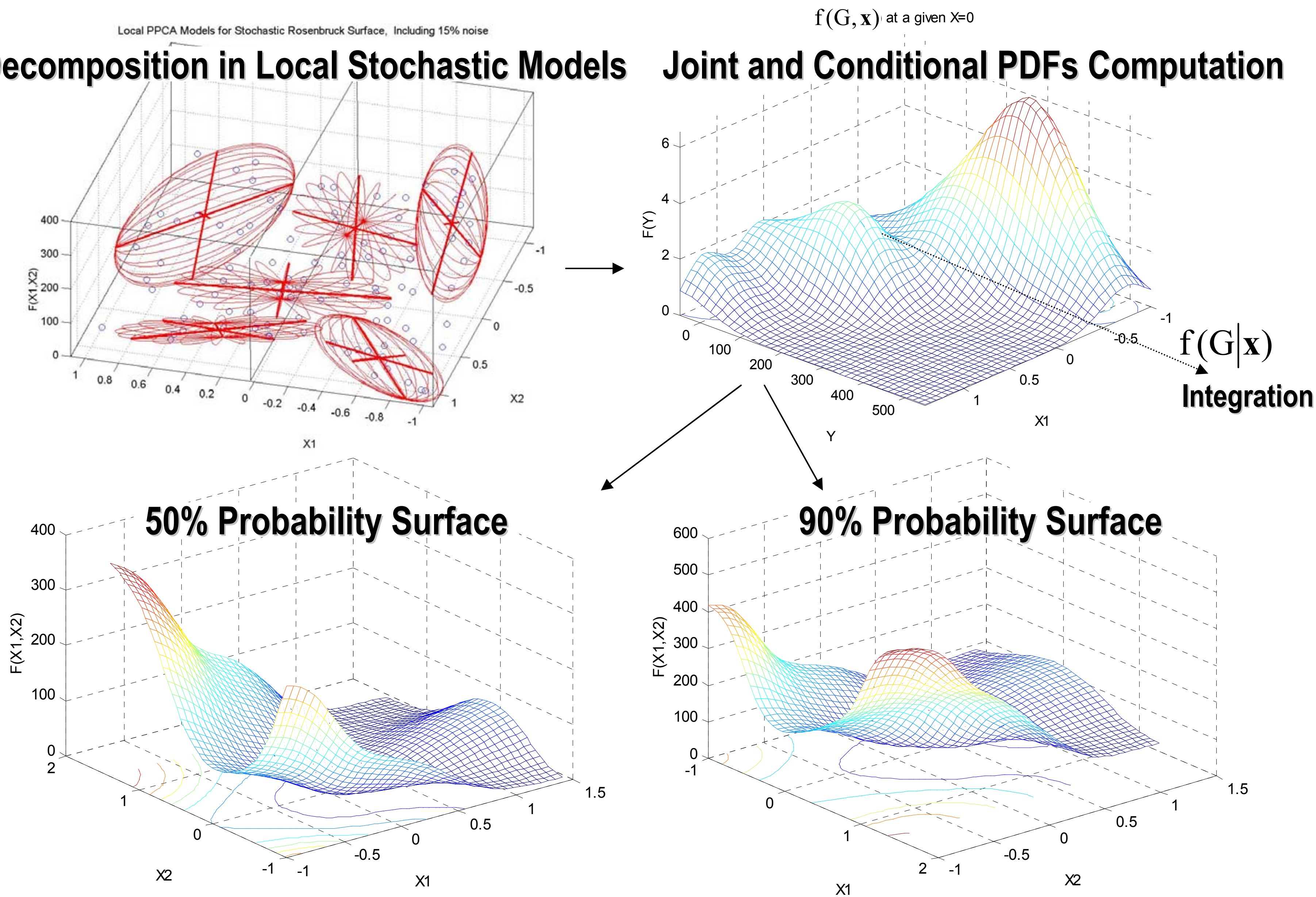
Computation of Probability-Level Response Surfaces



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Stochastic vs. Fuzzy Approximation

Stochastic Approximation

Stochastic 2L HM Approximation

$$E[y|\mathbf{x}] = \sum_{j=1}^{M} y_j(\mathbf{x}) h_j(\mathbf{x}) = \sum_{j=1}^{M} y_j(\mathbf{x}) \frac{\prod_i f(x_i|s_j) f(s_j)}{\sum_{i=1}^{M} \prod_i f(x_i|s_i) f(s_i)}$$

Priors are assumed to be independent standard Gaussian PDF

Fuzzy Approximation

$$f(x) = \sum_{j=1}^{M} y_j b_j(x) = \sum_{j=1}^{M} y_j \frac{\prod_{i=1}^{n} a_i f_i^j(x_i)}{\sum_{j=1}^{M} \prod_{i=1}^{n} a_i f_i^j(x_i)}$$

Fuzzy BF Approximation

Singleton fuzzifier, Gaussian membership functions, center average defuzzifier and product-inference rule

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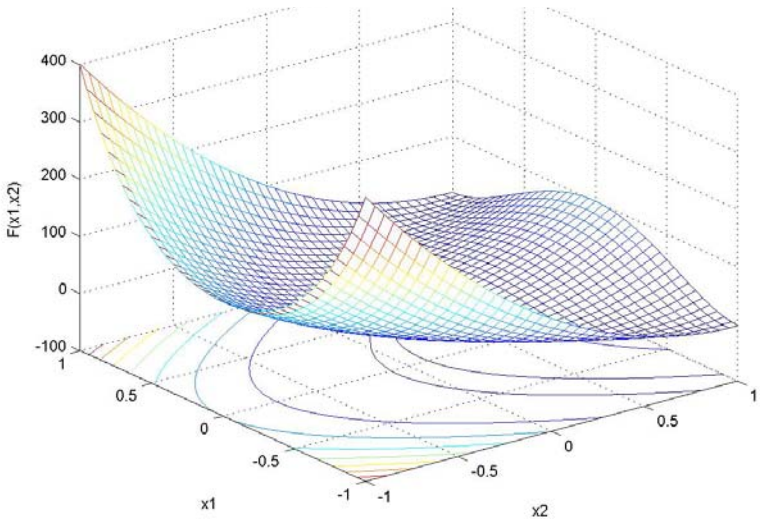
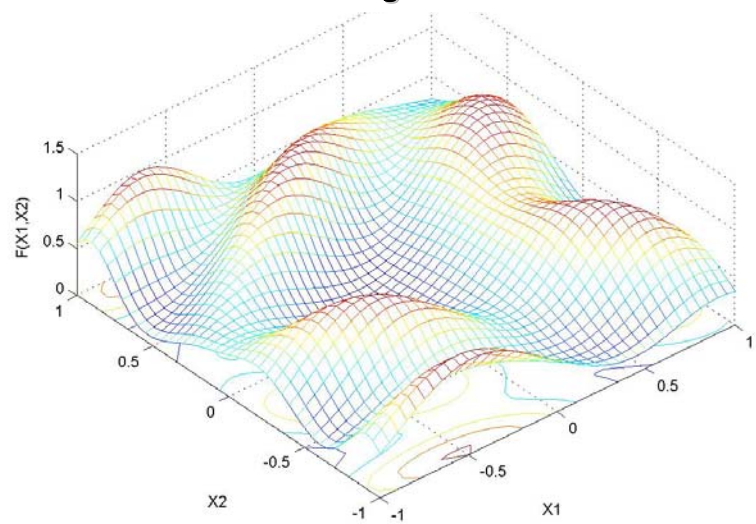
Fuzzy Clustering-Based Local BF Models

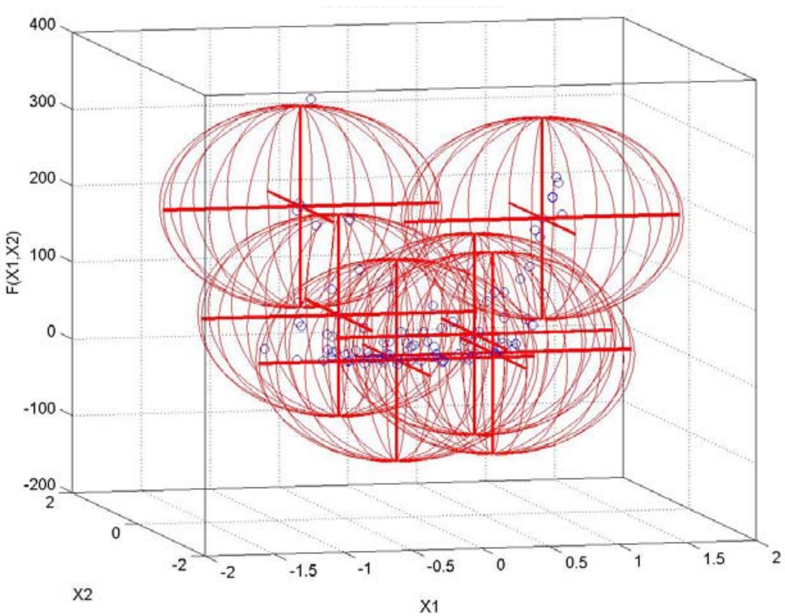
Smoothed Response Surface



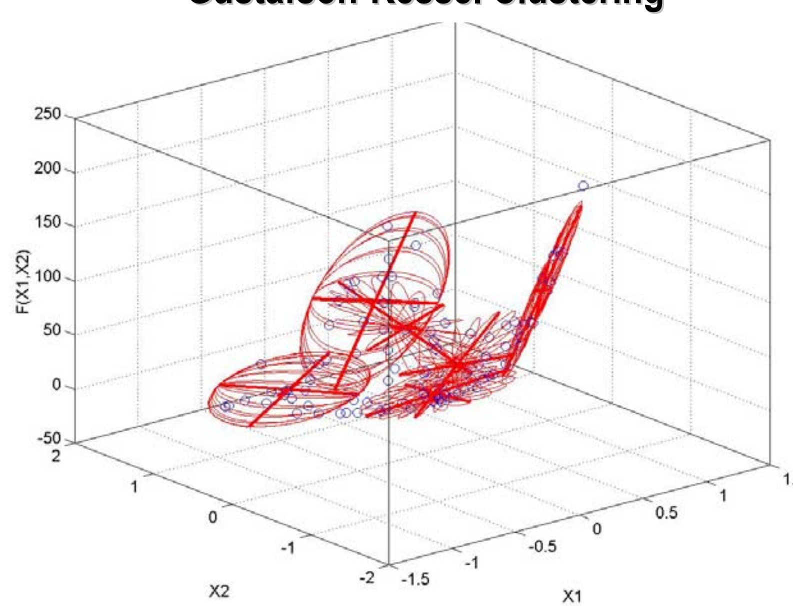
Summation of Unweighted Local JPDFs for X



Substractive Clustering



Gustafson-Kessel Clustering



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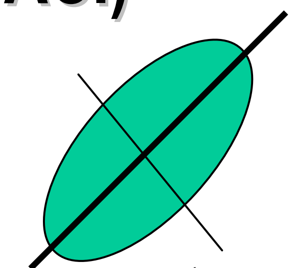
Euristic Stochastic Field Interpolation Schemes

Weighted Average Constant Interpolation (WACI)

$$E[y|\mathbf{x}] = \bar{y}(\mathbf{x}) = \frac{\sum_{i=1}^{NC} E[y_i] f(\mathbf{x}|s_i)p(s_i)}{\sum_{i=1}^{NC} f(\mathbf{x}|s_i)p(s_i)} = \frac{\sum_{i=1}^{NC} \bar{y}_i f_i(\mathbf{x},\mathbf{x}_i)p_i}{\sum_{i=1}^{NC} f_i(\mathbf{x},\mathbf{x}_i)p_i}$$

$$E[y|\mathbf{x}] = \bar{y}(\mathbf{x}) \quad \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} E_{i,j}[y|\mathbf{x}] f_{i,j}(\mathbf{x})\mu_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{m} f_{i,j}(\mathbf{x})\mu_{i,j}} = \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{y}_{i,j} h_{i,j}(\mathbf{x})$$



$\mathbf{a}_i$ = slopes

b_i = intercept

$$\mathbf{a}_i = -\frac{1}{\phi_i^y}\mathbf{\Phi}_i^x = -\frac{1}{\phi_{i,p+1}}\left[\phi_{i,1}, \phi_{i,2}, \ldots, \phi_{i,p}\right]^T$$

$$b_i = \frac{1}{\phi_i^y}\mathbf{\Phi}_i^T \bar{\mathbf{z}}_i$$

Weighted Average Linear Interpolation (WALI)

$$E[y|\mathbf{x}] = \bar{y}(\mathbf{x}) = \frac{\sum_{i=1}^{NC} y(\mathbf{x}_i) f_i(\mathbf{x},\mathbf{x}_i)p_i}{\sum_{i=1}^{NC} f_i(\mathbf{x},\mathbf{x}_i)p_i} = \frac{\sum_{i=1}^{NC} (\mathbf{a}_i^T\mathbf{x} + b_i) f_i(\mathbf{x},\mathbf{x}_i)p_i}{\sum_{i=1}^{NC} f_i(\mathbf{x},\mathbf{x}_i)p_i}$$

$$E[y|\mathbf{x}] = \bar{y}(\mathbf{x}) \quad \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} E_{i,j}[y|\mathbf{x}] f_{i,j}(\mathbf{x})\mu_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{m} f_{i,j}(\mathbf{x})\mu_{i,j}} = \sum_{i=1}^{n}\sum_{j=1}^{m} (\mathbf{a}_{i,j}^T\mathbf{x} + b_{i,j}) h_{i,j}(\mathbf{x})$$

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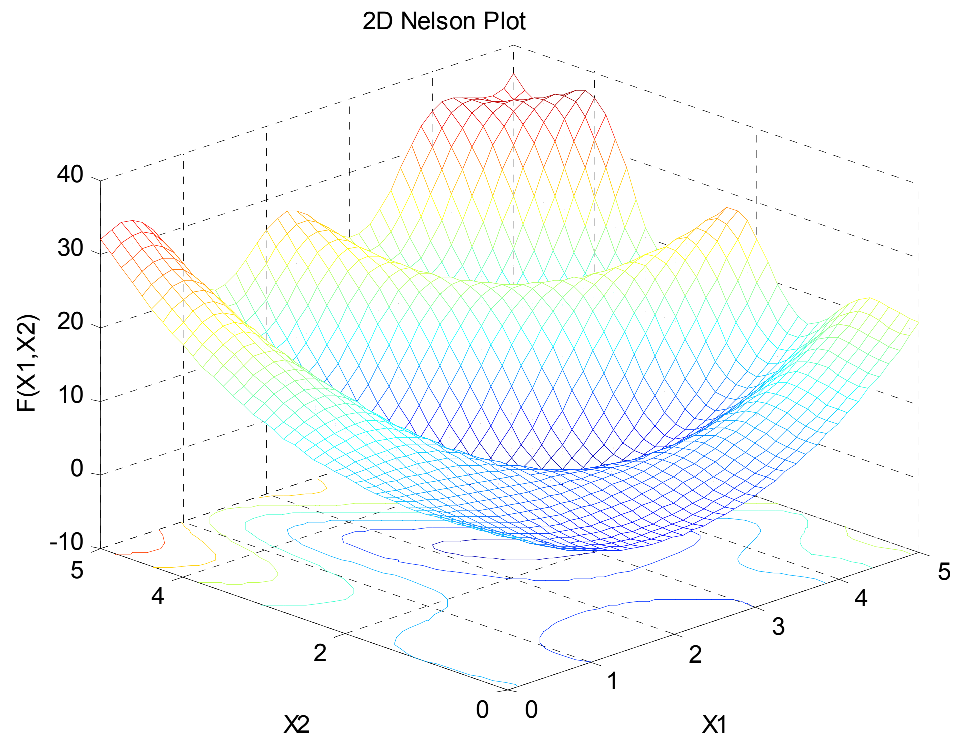
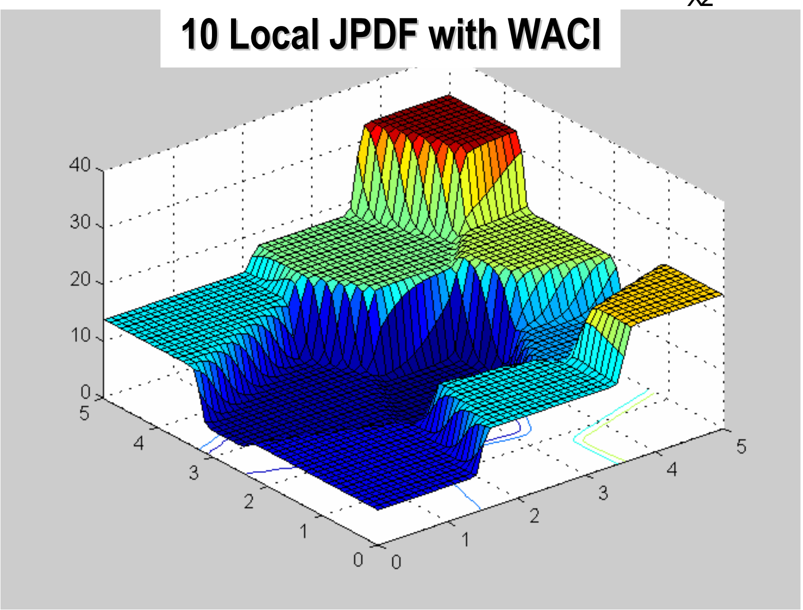
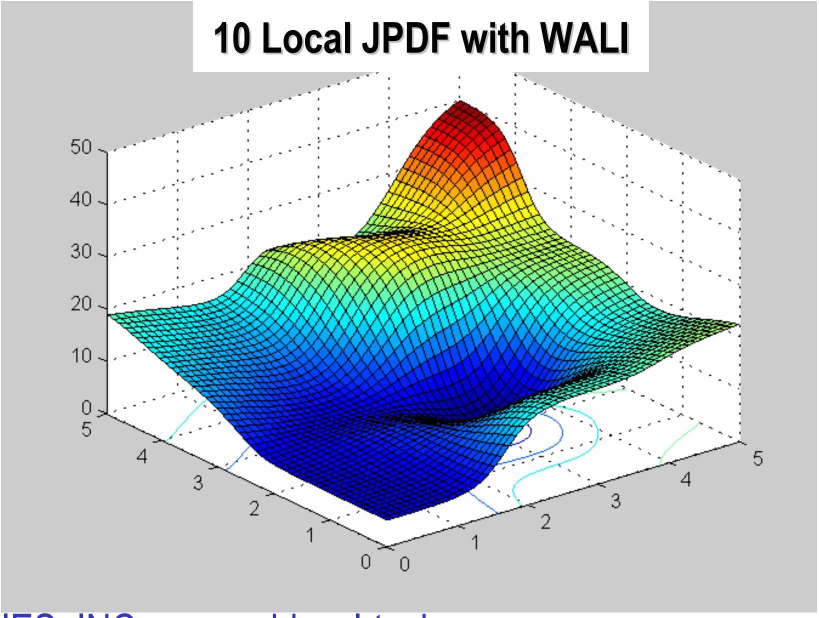
Nelson Surface







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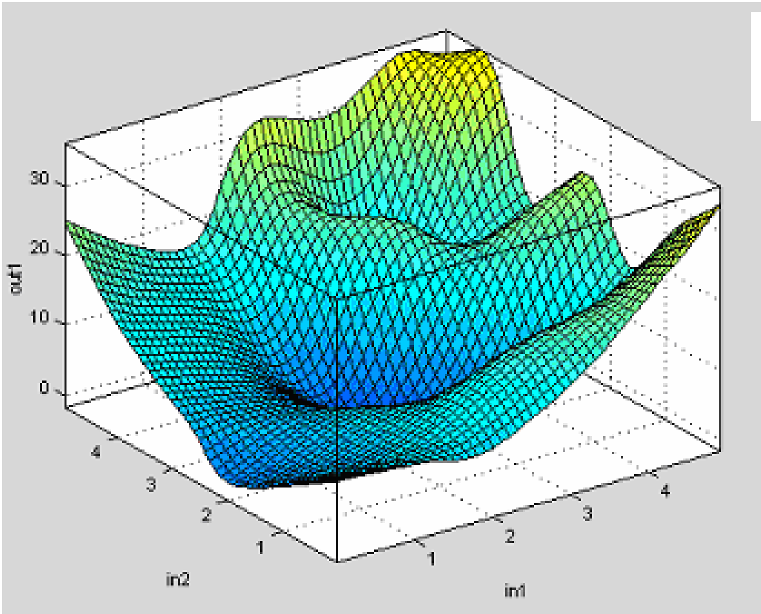
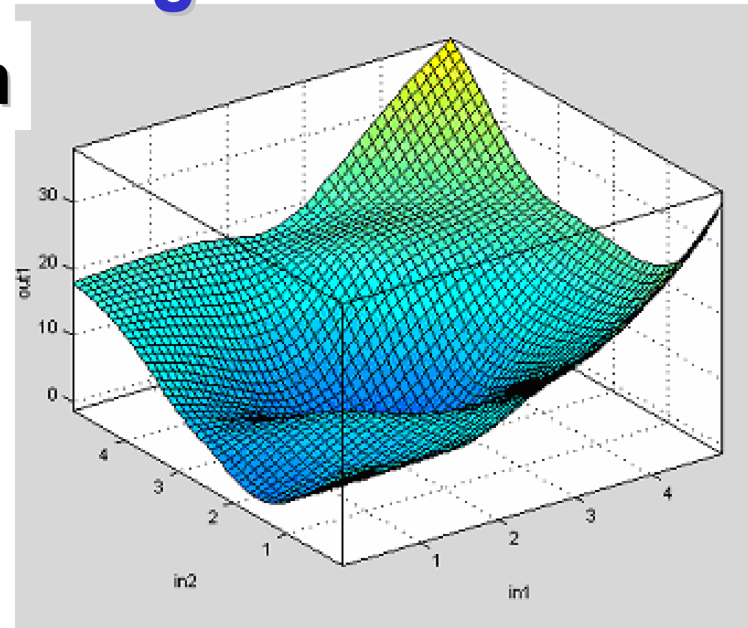
Substractive Clustering with WALI

Nelson

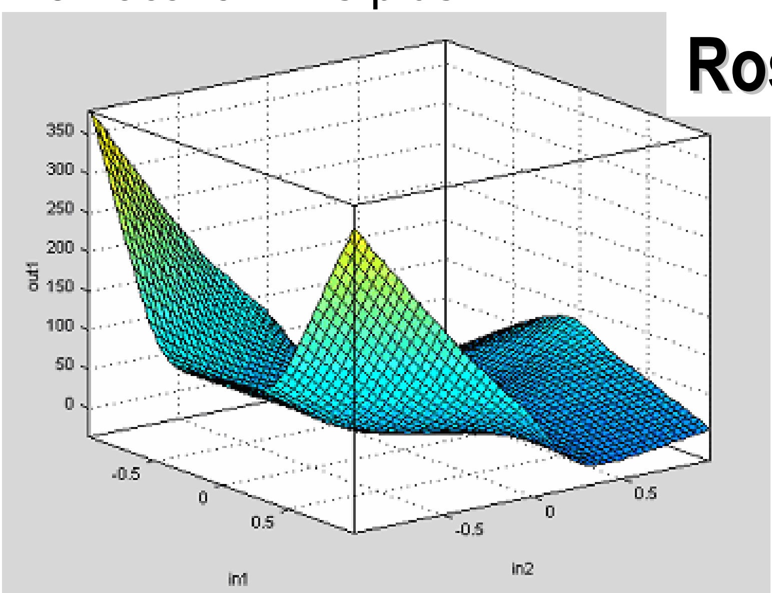


13 Local JPDFs plus WALI

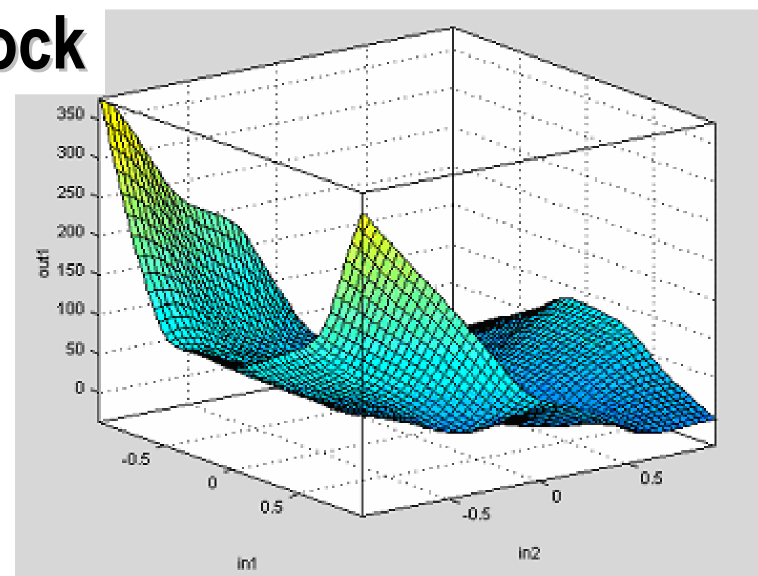


7 Local JPDFs plus WALI

Rosenbrock



8 Local JPDFs plus WALI



14 Local JPDFs plus WALI

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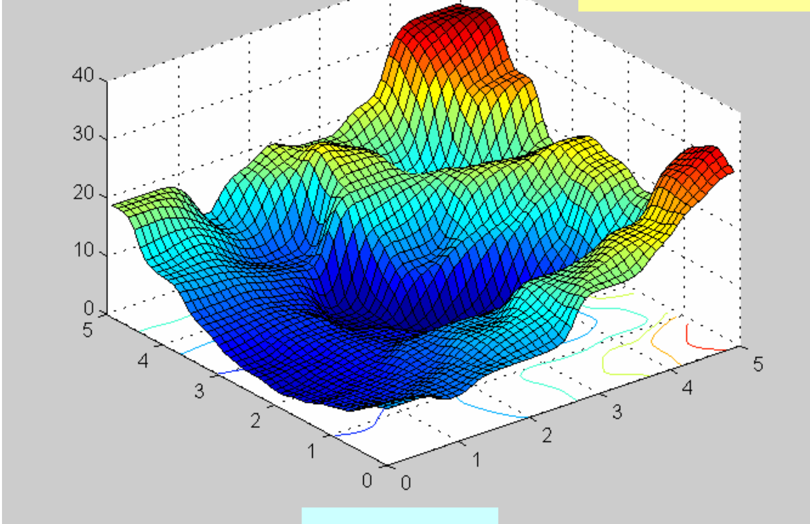
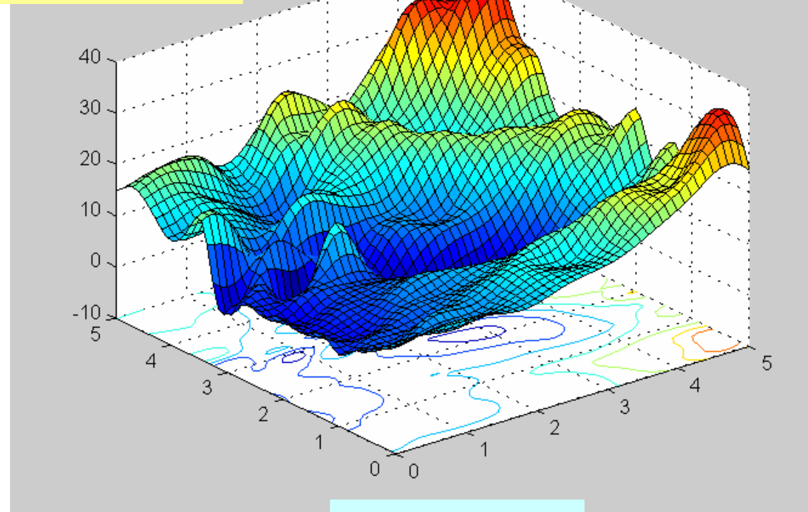
Two-Level Hierarchical Model Versus Krigging with 10% Noise

50% Probability Surface

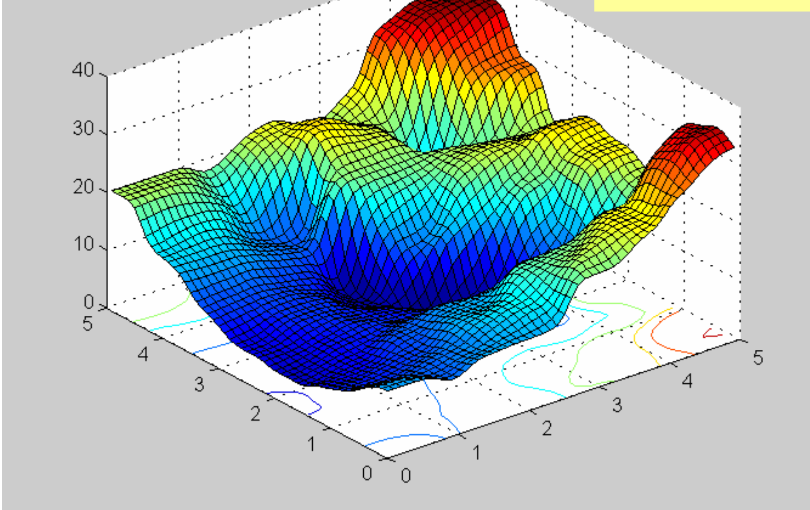


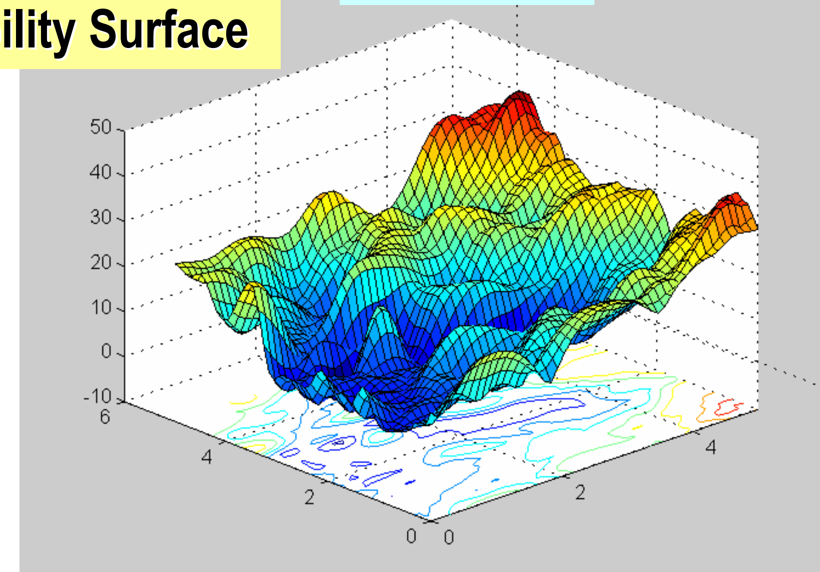


2L HM

Krigging

95% Probability Surface





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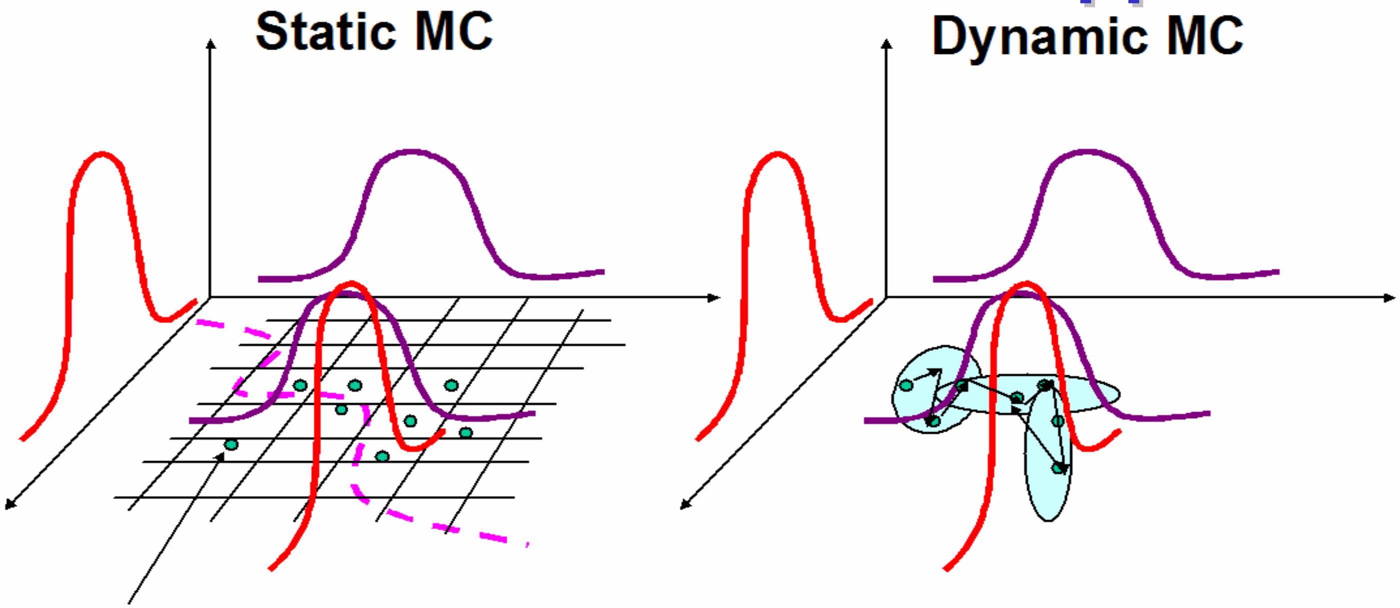
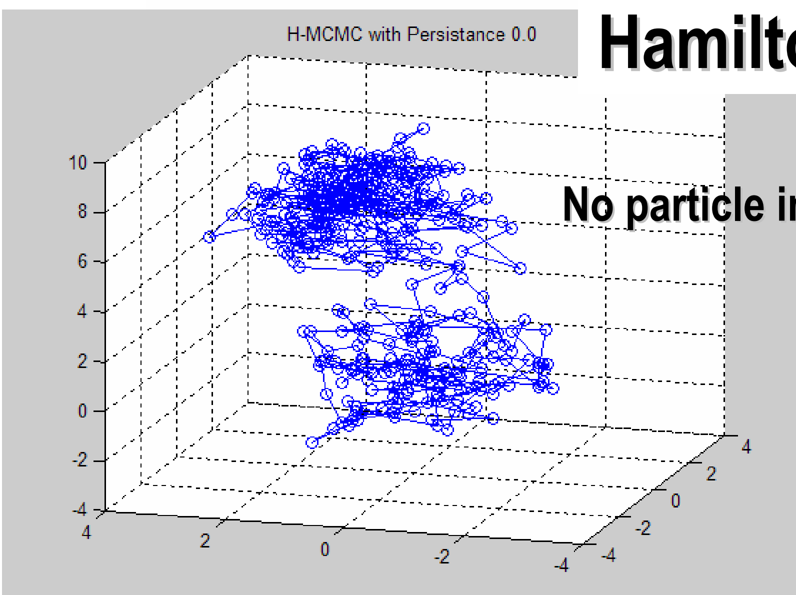
Dynamic MC for Stochastic RS Approximation

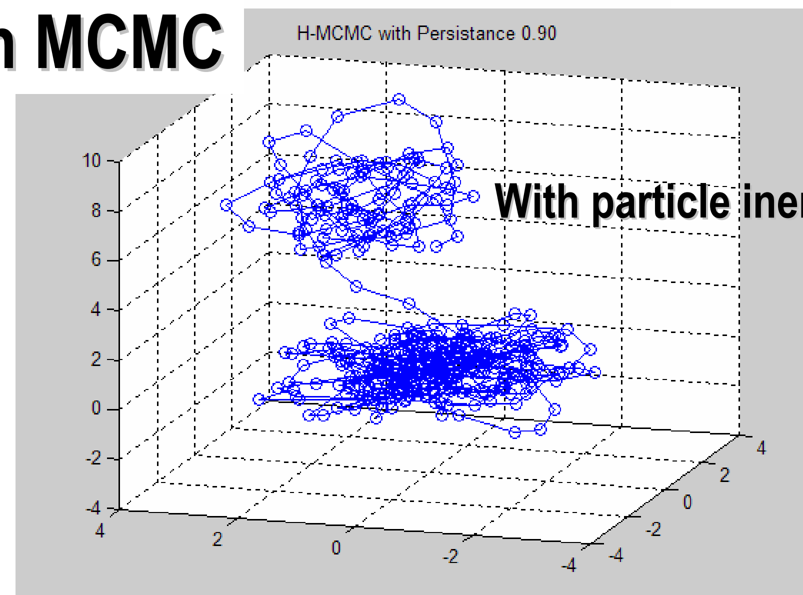


Sampling in *cartesian* space Sampling in *"state"* space

Hamiltonian MCMC



No particle inertia



With particle inertia

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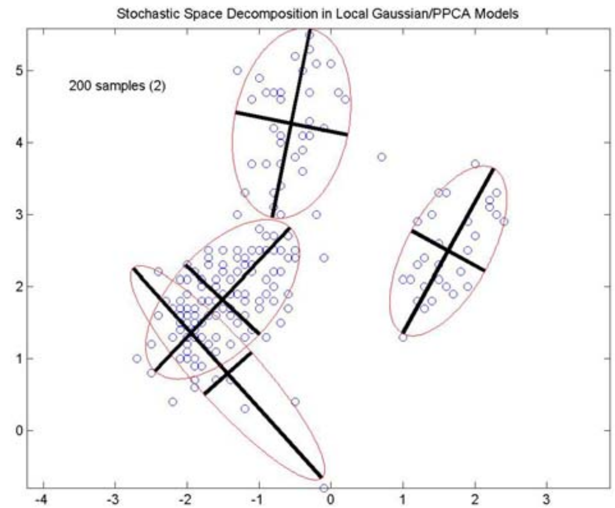
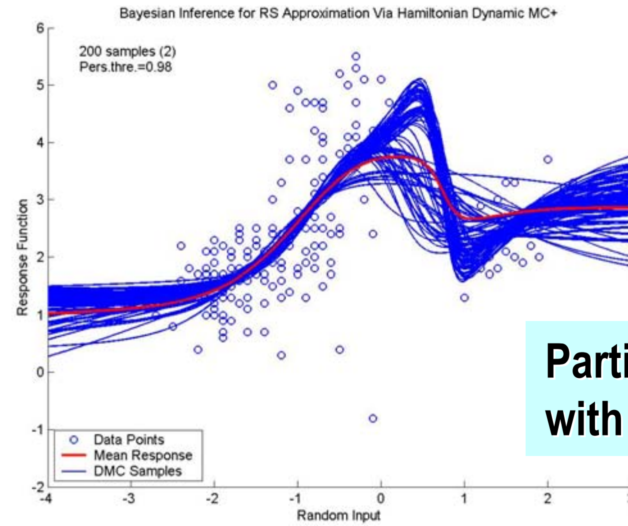
MCMC-Based Response Surface Approximation

Data and Local Models

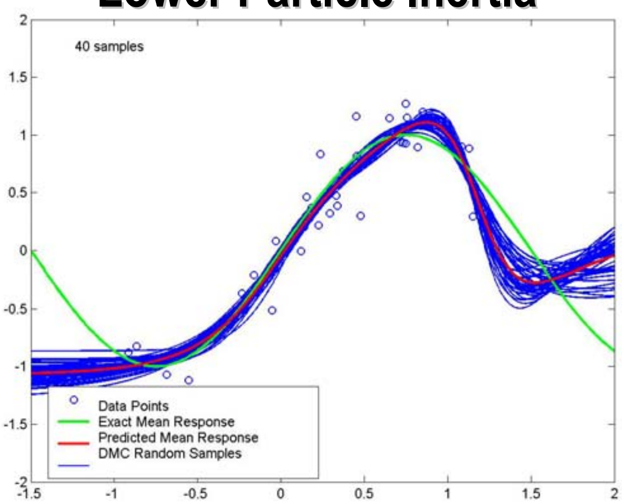


Simulated Conditional Mean Surfaces

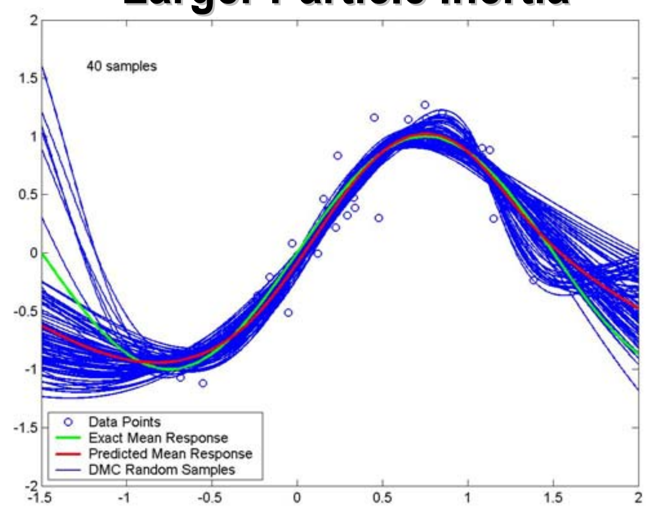


Particles with inertia

Lower Particle Inertia



Larger Particle Inertia



Remark: Confidence Intervals Depend on Local Data Density and Particle Inertia!

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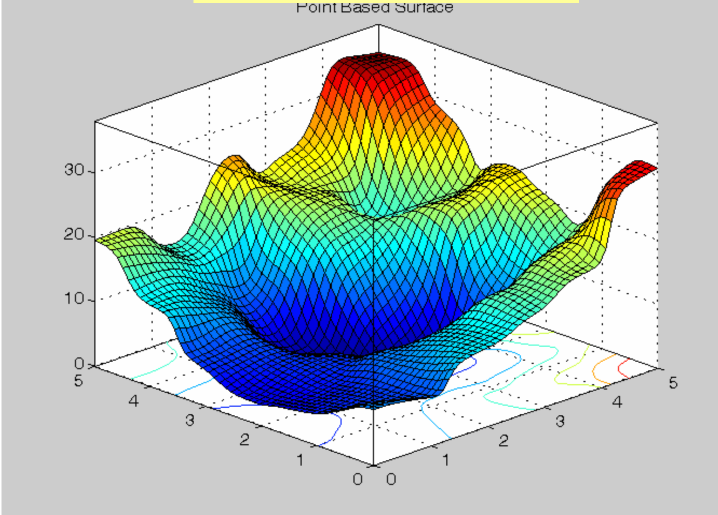
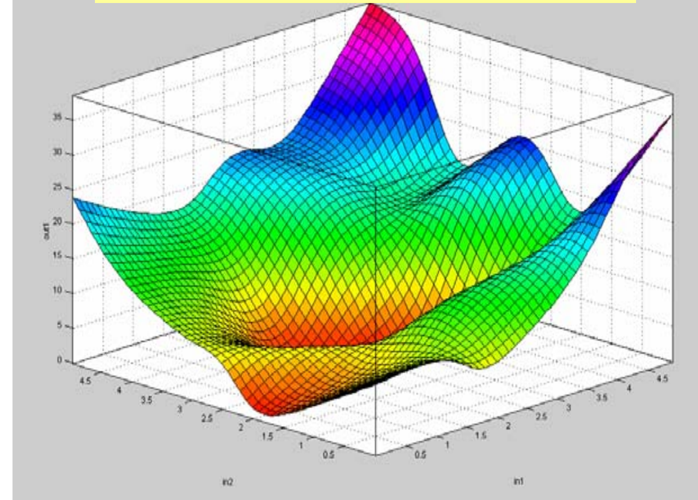
Comparison of 2L and 3L (Bayesian) HM Models

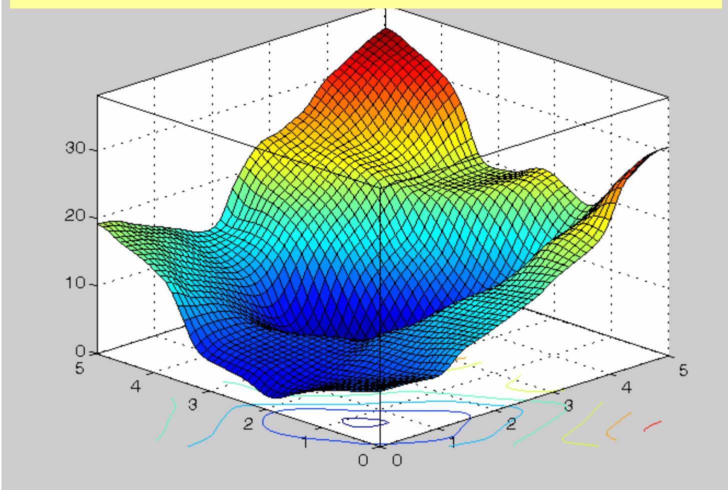
2L HM Point BF



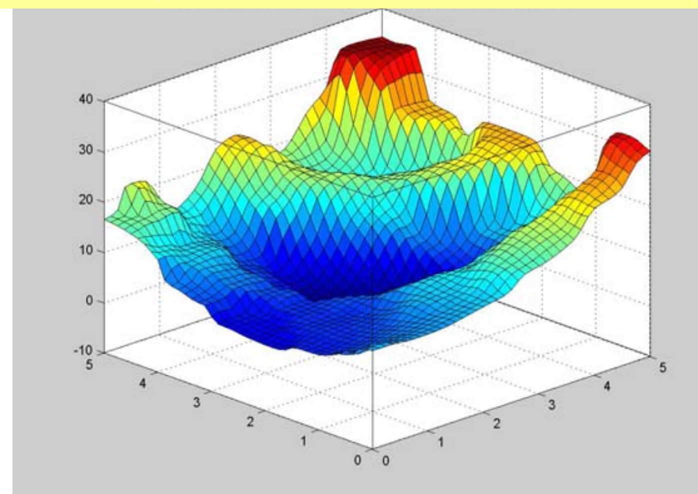
ANFIS 10 Subs Clust BF



MCMC-based 2L HM 10 Clust BF



3L HM (10 Clust BF with Point BF)



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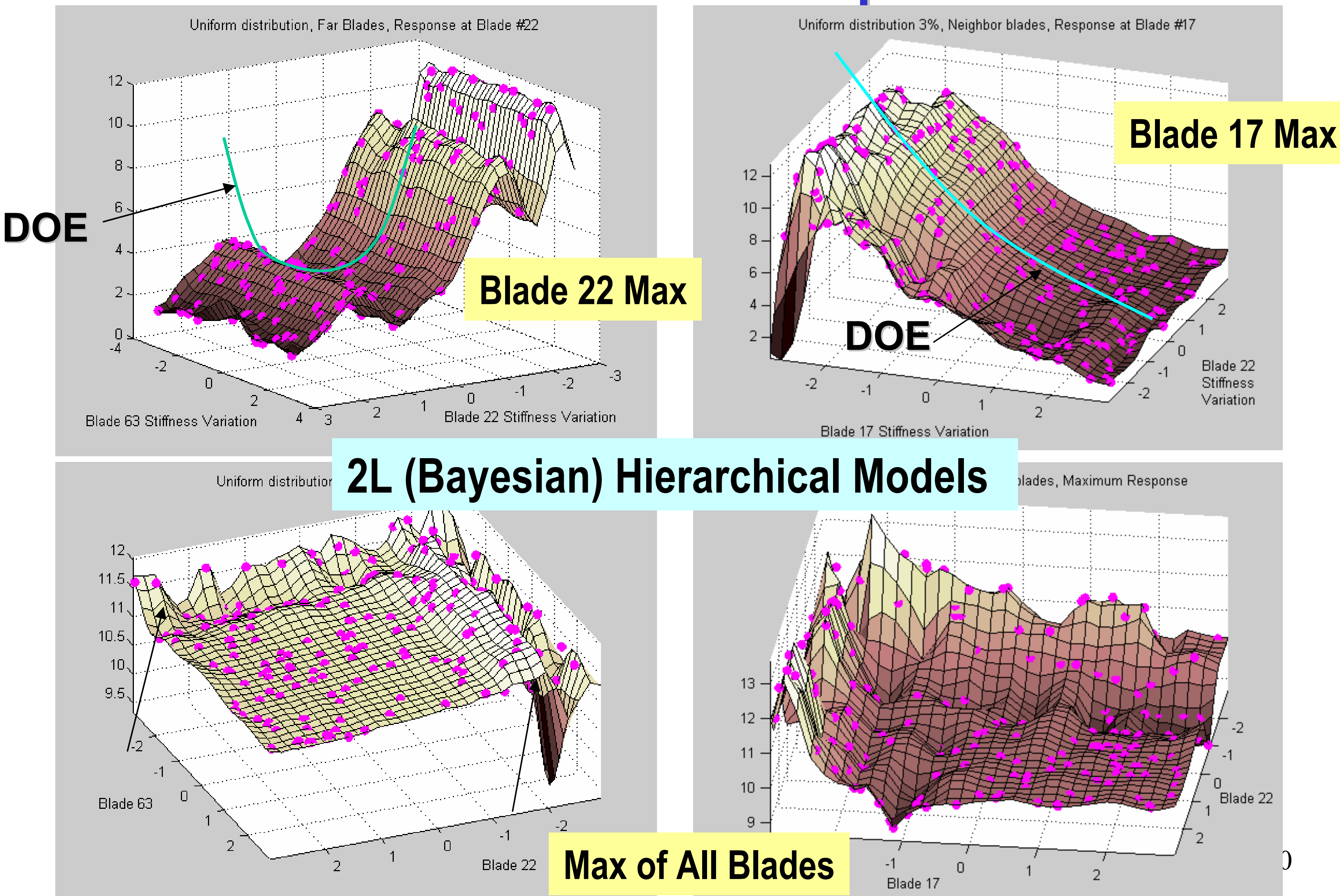
3. Application Examples

72 Blade Mistuned Blisk Response Surface



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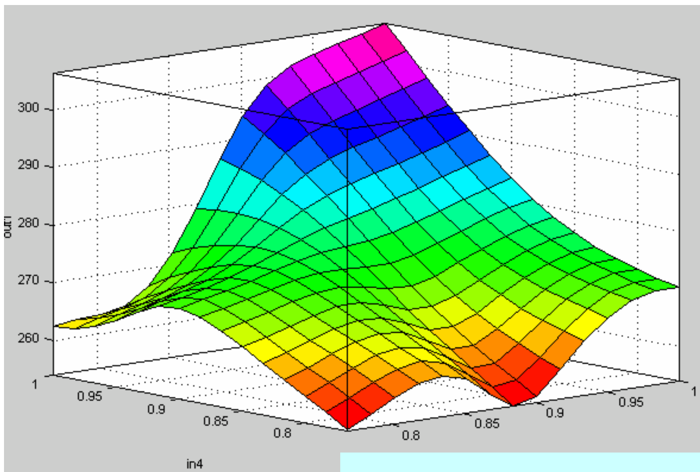
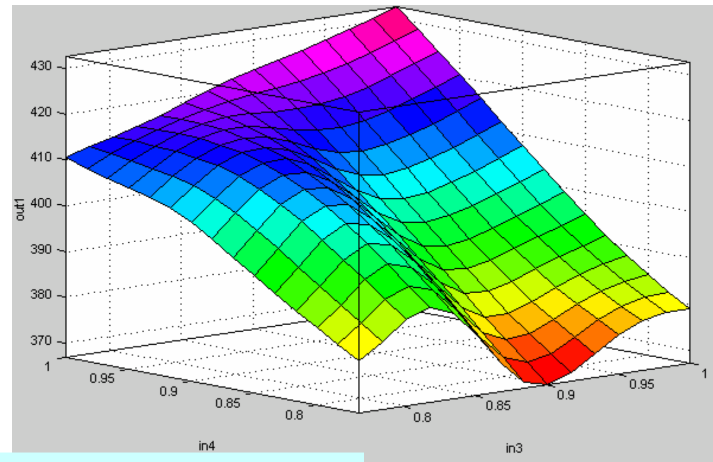
Engine Compartment Pressure Response Surface

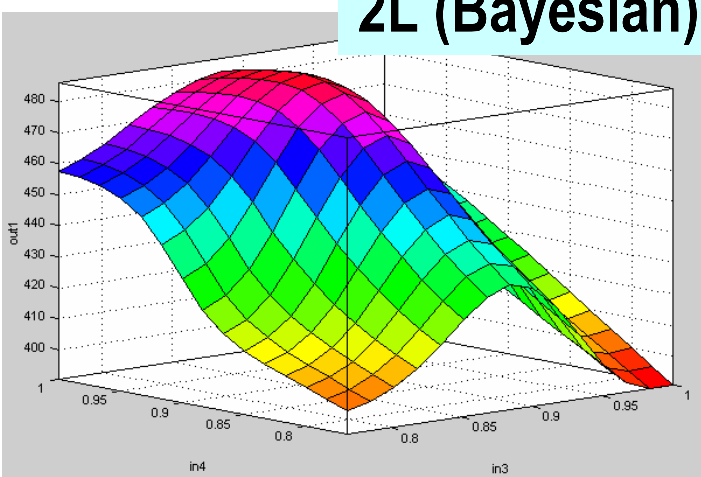
5D Highly-Nonlinear Response Surface

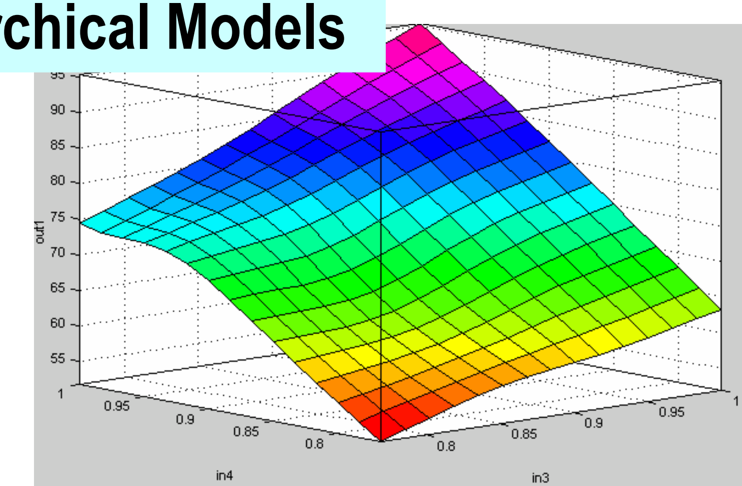
Surface Sections As Functions of X1, X2 (fixed values for X3, X4, X5)





2L (Bayesian) Hierarchical Models





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4. Concluding Remarks

1. Hierarchical stochastic approximation models, such as 2L and 3L HMs, are accurate tools for response surface modeling for complex stochastic problems.

2. Using HMs, probability-level response surfaces can be easily computed. They save large computational efforts in RBDO analyses.

3. The best stochastic approximation results were obtained using the proposed 3L HM that combines a pair of two 2L HMs.

4. The current practice approaches based using quadratic regression with DOE sampling rules can be inadequate for complex stochastic responses, as illustrated herein (slide 20).

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