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Novel Stochastic Concepts and Algorithms Integrated Within A Graphically-Assisted Reliability Prediction System for Aircraft Jet Engine Rotating Assemblies

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ABSTRACT

The paper discusses key aspects of probabilistic modeling for computing risks in engine bladed-disk systems. The paper focuses on the description of a integrated prototype software that has been recently developed by GP Technologies in collaboration with Pratt & Whitney and GE Aviation for predicting risks in Integrally Bladed Rotors (IBRs). The prototype software uses a refined stochastic modeling of blade geometry variations due to manufacturing that is integrated with a probabilistic forced response analysis capability using the ANSYS finite element code. The HCF risks are computed using a probabilistic Goodman diagram model. The prototype software incorporates Bayesian updating techniques that are employed to adjust probabilistic IBR stress model based on available test data. The effect of modeling (epistemic) uncertainties due to the limited number of stress samples (solution data) on IBR risk prediction is also considered.

INTRODUCTION

Predicting engine reliability is a critical aspect for both new engine design and operation of existing engine fleet. The new edition of the USAF Engine Structural Integrity Program (ENSIP) manual states that "*The probability of failure due to high cycle fatigue (HCF) for any component within or mounted to the engine should be below 10^{-7} per EFH on a per-stage basis, provided the system-level safety requirements are met*".

The new ENSIP requirements for probabilistic design produce *a critical need* for developing an integrated risk prediction software with an open architecture. To respond to these new requirements of engine technology, GP Technologies, Inc., in collaboration with Pratt & Whitney and GE Transportation, has developed a *graphically-assisted, object-oriented probabilistic bladed-disk design prototype software that incorporates state-of-the-art stochastic modeling and simulation algorithms integrated with physics-based computational engineering design tools, test and field databases*. The prototype software incorporates advanced Bayesian inference techniques, stochastic response approximation models, stochastic simulation algorithms, and Bayesian techniques for probabilistic model updating and for incorporating modeling uncertainties due to limited sample data.

The paper discusses some of the key stochastic modeling features that are incorporated in the IBR risk prediction prototype software. These features are illustrated by using a simple full 3D IBR ANSYS model.

The prototype software, in addition to state-of-the-art stochastic tools for modeling, simulation and updating, includes also a set of innovative, new algorithms. These new, innovative algorithms include (i) stochastic modeling and simulation of blade geometry deviations based on a double expansion Karhunen-Loeve expansion model (Ghiocel, 2004) capable of incorporating both the within-blade and inter-blade spatially correlated variations with highly non-Gaussian distributions, (ii) three-level hierarchical models for IBR stochastic response approximation, and (iii) new stochastic reduced-order modeling for random mistuning using either an eigen (Ghiocel, 2002) or a Krylov subspace projection scheme (Ghiocel, 2005) and that can incorporate the effects of random blade geometry variations on blade frequencies and mode shapes. Some of these new, innovative algorithms are described elsewhere as above referenced, and some are proprietary information.

To be fully practical and used efficiently by engine designers, the prototype software is developed in extremely user-friendly environment. To achieve the user-friendliness goal, the prototype software employs various "graphically assisted" modeling tools using the SIMULINK/MATLAB computing environment.

SIMULINK provides a powerful interactive graphical user interface that is used in building graphically block diagrams, flowcharts, performing simulations, as well as analyzing results. Using SIMULINK, we developed computational models that are hierarchical, so that a system can be viewed at high level, but also at intermediate levels, down to the basic component level. A great advantage is that SIMULINK is intimately integrated with MATLAB, providing immediate access to an extensive range of tools for algorithm development in any programming language, data visualization, data analysis and access, and numerical computation. The user can quickly create new model, and maintain a detailed block diagram of a system using a comprehensive set of predefined blocks. Using the SIMULINK environment, one can build computational toolboxes and models by dragging and dropping blocks from the library browser onto the graphical editor and connecting them with lines that establish mathematical relationships between the blocks. The analyst has immediate access to common graphical editing functions, such as copy, paste, and undo.

The SIMULINK/MATLAB-based Probabilistic Bladed-Disk Design prototype software is based on a set of basic computational blocks or graphical objects that can be integrated quickly in an object-oriented system. Currently, in the SIMULINK-based prototype software four different analysis options are implemented:

1) *Preliminary Deterministic Nominal Geometry IBR Forced Response Analysis*. This includes a single SIMULINK preconfigured blockset and a graphical user interface for plotting specific results of the ANSYS analysis.

2) *Probabilistic IBR Forced Response Analysis*. This includes two SIMULINK preconfigured blocksets for i) Stochastic Blade Geometry Modeling and Simulation and ii) Probabilistic IBR Forced Response Computation. A graphical user interface for plotting results of probabilistic ANSYS analysis has been also developed.

3) *Deterministic IBR Forced Response Analysis for selected geometry variations*. This includes two SIMULINK blocksets for two different analysis options: (i) compute mean-geometry IBR response and (ii) selected measured or simulated random-geometry system. It also includes a graphical user interface for plotting results.

4) *IBR Risk Prediction.* This includes only a set of algorithms and a graphical user interface for plotting the reliability analysis inputs and results. The user interface includes various plots of the stress (steady and vibe stresses) probabilistic distribution at critical location and probabilistic Goodman diagram, including also results with *Bayesian Updating* of computed probabilistic stresses based on testing data and *Modeling Uncertainty* for computing variation bounds (confidence intervals) of predicted risks.

Figure 1 shows the SIMULINK blockset modeling for *Probabilistic IBR Forced Response Analysis* using the ANSYS code.

STOCHASTIC BLADE GEOMETRY MODELING

Stochastic blade geometry variation modeling is a key aspect for obtaining accurate probabilistic IBR forced response predictions. The accuracy of stochastic modeling of blade geometry variations due to manufacturing process impacts directly on the accuracy of IBR system response. Structural mistuning, aero-forcing, aero-damping, blade flutter that are complex random phenomena that can affect drastically the IBR forced response, are sensitive to very small variations in blade geometries. Both blade mode shapes and unsteady pressures on blade surfaces are varying significantly depending on the spatial variation pattern of blade geometry variations. Moreover, the blade manufacturing variations are extremely complex with highly non-stationary spatial variations on the surface in both chord and span directions. The complexity of the blade geometry variations and the importance of their accurate modeling for predicting correctly the IBR forced response require refined stochastic spatial variation approximation models.

In the prototype software, the IBR geometry variations due to manufacturing are modeled using 3D-3V stochastic field models (three measured variations in three spatial coordinates, x, y and z) based on CMM blade data. Two types of stochastic field models were implemented to idealize blade geometry variations:

(1) *Covariance-based expansion models (one-level hierarchical models)* These stochastic field models are based on (Ghiocel, 2004): (i) Karhunen-Loeve (KL) expansion, or equivalently Principal Component Analysis (PCA) expansion or Propper Orthogonal Decomposition (POD) expansion, and (ii) Choleski decomposition.

    The two covariance-based expansion models have been implemented in original and transformed data space (as translation fields) and are capable of including highly non-Gaussian probability distributions. The KL/POD/PCA expansion models have the advantage of reducing the complexity of geometry variations to a reduced number of *manufacturing mode* shapes with statistical coefficients, typically with a non-Gaussian distribution (Ghiocel, 2004, Cassenti, 2003, Griffiths and Tschopp, 2004).

(2) *Joint PDF-based expansion models (two-level and three-level hierarchical models).* These stochastic models capture complex spatial variability patterns with highly non-Gaussian variation directly in original data space (Ghiocel, 2004).

Three types of blade statistical databases were considered (currently operational): (i) only measured rotor database, (ii) only stochastically simulated rotor database and (iii) combined measured and simulated database. Also, the implemented rotor geometry database formats can include: (i) single blade measurement, (ii) multiple blade or rotor measurements without considering the blade location effects, or in other words without considering the spatial correlation between inter-blade geometry variations, and (iii) multiple rotor measurements including the blade location effects, i.e. inter-blade variation correlations.

The options currently available for stochastic modeling of blade geometry variations due to manufacturing are:

(i) *Simple Blade Model (SBM).* Stochastic blade geometry variations are assumed to be a Gaussian stochastic field with a *homogeneous and quadrant-symmetric* spatial correlation structure.

(ii) *Refined Blade Model (RBM).* Stochastic blade geometry within-blade variations are assumed to be non-Gaussian stochastic fields with *non-homogeneous and anisotropic* spatial correlation structure variation. The inter-blade variations are assumed to be statistically independent.

(iii) *Refined Rotor Model (RRM).* Stochastic blade geometry within-blade variations are assumed to be non-Gaussian stochastic fields with *non-homogeneous and anisotropic* spatial correlation structure variation. The inter-blade variations are assumed spatially correlated based on the covariance matrix of inter-blade variations.

The prototype software uses an extremely efficient subspace projection scheme (proprietary algorithms) for computing spatially correlated inter-blade geometry variations. *Without this new stochastic reduced-order modeling, the full-rotor covariance matrix sizes become impractical.*

For example, for a 100 blade IBR with a mesh of 200 measurements points in 3D space with x, y and z coordinates, the covariance matrix has to be computed, for example, for a grid of 100 x 200 x 3 points that means it has a size of 60,000 x 60,000 with a total of 3,600,000,000 elements. The new stochastic reduced-order modeling reduces the entire full-rotor covariance matrix to two covariance matrices, one for within-blade variation with a size of 600 x 600 with only 360,000 elements and one for inter-blade variations that for 100 blades x 5 KL modes per blade has a size of 500 x 500 with only 250,000 elements. Thus, the reduction in array storage is thousands of times, and the reduction in computing time for KL modes is in the range of tens of thousand times.

Figures 2 and 3 show computed covariance matrices for within-blade and inter-blade geometry variations. While trying to understand these plots, it should be noted that the within-blade covariance matrix is computed for a 3D stochastic variation in x, y and z directions (it has 3 x 3 = 9 partitions), while inter-blade covariance matrix is computed for a 1D stochastic variation defined by the KL mode coefficients (it has single partition). It should be noted that the inter-blade variation covariance matrix is almost diagonal except for a few of blade locations that indicate a significant spatial correlation both positive and negative.

Figures 4 and 5 show the blade geometry variations for a given variation and a simulated variation, respectively. It should be noted that the blade variation is described by a 3D stochastic field with a quite complex, non-stationary pattern (requires an anisotropic covariance option) that indicates larger-wavelength fluctuations along the airfoil chord and shorter-wavelength along the span.

STOCHASTIC FORCED RESPONSE ANALYSIS

To perform the probabilistic IBR structural analysis of a geometrically-mistuned system, the analyst needs to build first the nominal geometry model, and then input aero-loading and boundary conditions. For computing the probabilistic system forced response, the prototype code automatically maps the blade geometry deviations from the measurement data grid onto the refined computational grid, without changing the grid topology (for the deformed grids including random geometry variations). We used three stochastic interpolation schemes to map the blade measurements on the model grid geometry: (i) Gaussian Process interpolation model, equivalent to Gaussian Krigging interpolation ,(ii) Local shape function interpolation model (LSF) and (iii) Delaunay tessellation model (DT).

The computational steps required for performing a probabilistic forced response analysis using ANSYS are:

1) *Build the measured rotor geometry database* (in MATLAB format based on available data files)
2) *Simulate stochastic geometry rotor database* (can also include measured rotors)
3) *Build the reference, nominal finite element model*
4) *Prepare specific input data files for nominal and probabilistic analyses, including the definition of probabilistic models of aero-forcing and modal damping based on aero test data*
5) *Set the SIMULINK block parameters for deterministic and probabilistic analysis* (all these block parameters are self explained and easy to understand for the analyst user)
6) *Run SIMULINK-ANSYS deterministic analysis for the nominal geometry system*
7) *Update probabilistic inputs based on the nominal analysis results*
8) *Run SIMULINK-ANSYS probabilistic analysis for stochastically simulated rotors*
9) *Post-process graphically the analysis results in the MATLAB graphical environment*
10) *Generate automatically generated engineering reports based on user option selection*

Figures 6 and 7 show the same vibration mode computed for the tuned system and for a randomly mistuned system, respectively. Figures 8 and 9 show the mistuned Campbell diagram and a blade tip frequency response function (for an Engine Order equal to 13) computed for a randomly mistuned system.

The random variation of blade aero-forcing was modeled using a stochastic pressure uncertainty factor, a stochastic stimulus for each Engine-Order excitation and a random modal damping that can be different for each system mode families.

IBR RISK PREDICTION

The HCF risk prediction is implemented in the prototype software in two optional steps:

(1) Review computed steady-vibe stress pairs at user-selected or critical location (Figure 10). It should be noted that steady and vibe stresses are paired, since each stress pair (bivariate stress data) corresponds to a distinct simulated IBR geometry variation. This is more accurate than separating steady and vibe stresses in two one-dimensional models, and by this to lose their connection through the same IBR geometry variation.

(2) Compute HCF risk using stochastic simulation. To count the HCF failures, for each simulated stress point in steady – vibe stress space (bivariate data), we compare its random location with a simulated random Goodman diagram interaction curve.

Figure 10 illustrates the computed IBR steady-vibe stress data pairs and the associated estimated bivariate stress probability density (in 2D, for steady and vibe stresses) for 250 simulated stochastic geometry IBRs. Currently, for estimating the bivariate stress probability density function (PDF) we implemented a density estimation technique that produces a non-analytical probability distribution estimate based on a two-level hierarchical model (Ghiocel, 2004, 2005). This PDF approximation technique represents a significant advancement for computing IBR stress PDF estimates. Typically, OEMs consider separate one-dimensional analytical PDFs for steady and vibe stresses. There is no reason for IBR mistuned stresses to fit an analytical distribution. In fact in-house tests have indicated that mistuned vibe stresses are extremely sensitive to any small blade geometry changes that have a large impact on the shape and tails of their probability distributions.

Bayesian updating is used to adjust the computed stress bivariate PDF based on new evidence coming from test data (such as steady stresses and/or mistuned stresses determined by strain-gages and/or NSMS). For applying Bayesian updating, the analyst has to select the statistics to be considered as random hyper parameters for the probabilistic updating. Also, the analyst needs to provide the test data for updating of the computed stress probability density. The selected hyper parameters can be global statistics of the bivariate stress PDF, such as the mean and covariance function (variance in each stress space axis, plus cross-correlation), or local statistics of the hierarchical density models, such as local density statistics or weighting factors. We considered hyper parameter distributions that can be either normal, lognormal, Gamma or Beta.

Figure 11 shows results of the Bayesian updating for the research bladed-disk example. In this made example (not a real design), using test data (the filled green squares), the computed stress PDF (see the stress data points and the PDF contours that are on the right-side of the testing data) was updated. The updated stress PDF is shown by the PDF contours that surround the testing data points. The testing data show a shift in the mean toward lower steady stresses and a very small variation in vibe stresses. As a result, the prior computed HCF risk is 0.0724, while the updated HCF risk is only 0.0012. This indicates that for this made example, the computational probabilistic forced response analysis is very conservative since produces much larger steady stresses and very large and variable vibe stresses.

Figure 12 shows the effect of modeling uncertainties produced by the limited number of stress simulated samples on IBR predicted risks. In the shown example, 250 simulated data were considered. It should be noted that the HCF risk is a random variable (uncertain estimate) with a mean of 0.061 and a standard deviation of 0.016 (about 25% coefficient of variation). If the effect of limit simulation number is not considered, then for the same problem the reference HCF risk is 0.0724.

To avoid the large increase in computational effort due to nested loops, we developed an innovative simulation technique for assessing the effect of limited number of samples on IBR risk prediction. Instead of looping at the stochastic input level, we loop at the stochastic output level, and by this we avoid repeating computational mechanics analyses. This produces huge computational savings at no loss of stochastic prediction accuracy. The new technique is based on using fast Markov Chain Monte Carlo (MCMC) simulation directly from the simulated IBR stress data. The idea is that instead of assuming lack of data at stochastic input level, say for stochastic blade geometry model, we assumed lack of data at the output analysis level, say for the simulated IBR stress data. To do this we combined MCMC simulation with system particle dynamics using a variational Hamiltonian formulation. Starting from the information provided by the limited number of simulated IBR stress data points (or particles), we build the Hamiltonian functional associated to this data or particle systems. Then we free particles to move. After a sequence of repeated random moves, all particles are placed in new random locations. These new particle locations produce a set of data points that are randomly distributed in space but respect the underlying physics that is captured through the Hamiltonian functional of particle system. Importantly, this stochastic simulation technique for incorporating modeling uncertainties due to limited number of sample data is free of any user's assumption on the selection of probabilistic model hyper parameters and their distribution types.

SUMMARY

The paper describes a prototype software for computing IBR risks. The prototype software incorporates state-of-the-art stochastic modeling and simulation tools integrated physics-based engineering tools.

To achieve the software user-friendliness goal, the prototype code uses an advanced graphical object-oriented modeling. Refined stochastic field models are used for blade manufacturing variations.

Bayesian algorithms are used to integrate various types of information from analyses, tests and field observations, and to incorporate modeling uncertainties.

ACKNOWLEDGMENTS

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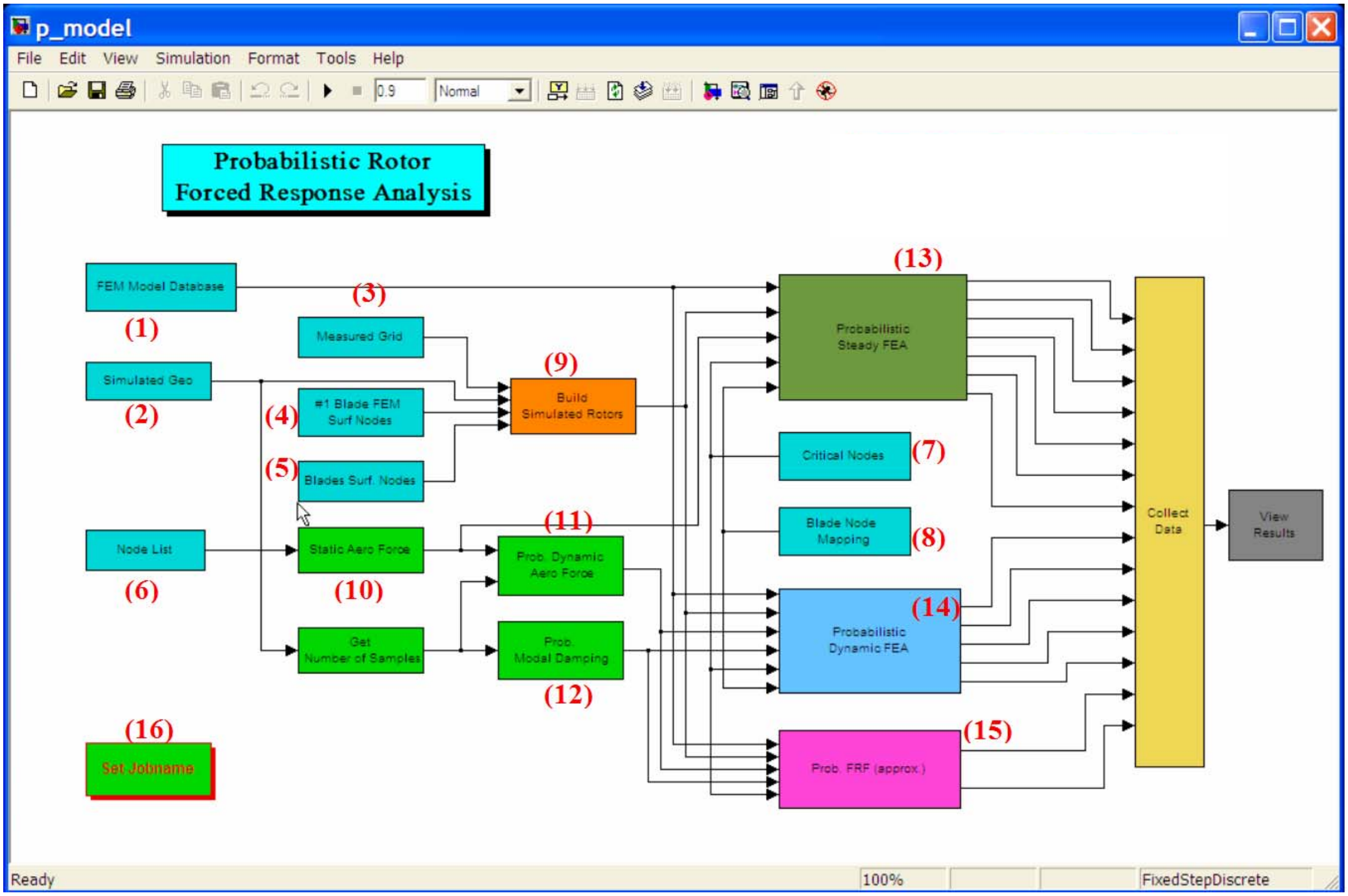

Figure 1 Probabilistic (Geometry Mistuning) IBR Forced Response Analysis Using Graphical Object-Oriented Modeling

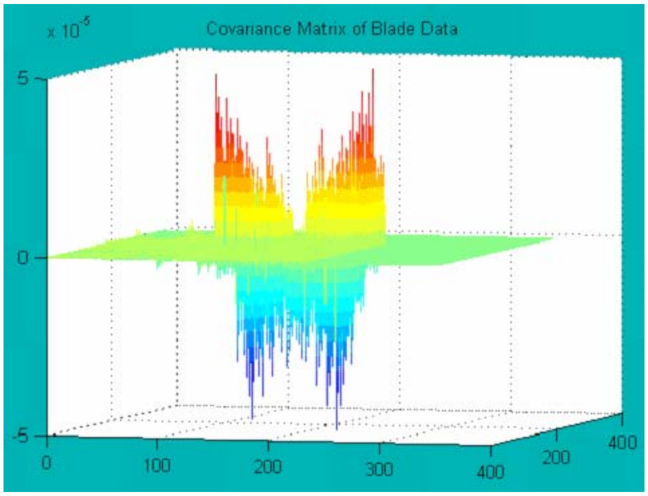

Figure 2 Within-Blade Variation Covariance Matrix

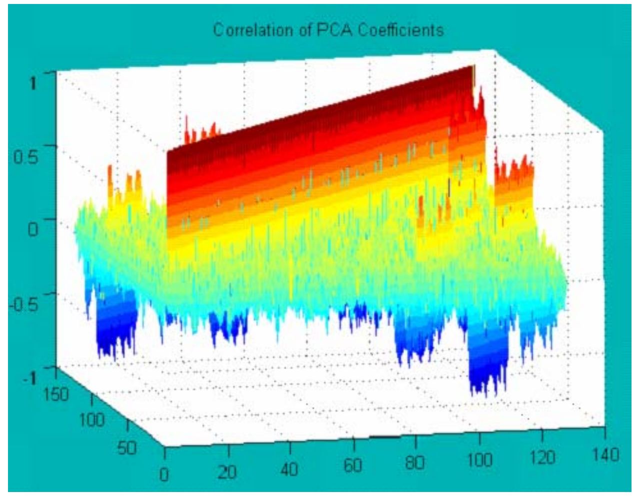

Figure 3 Inter-Blade Variation Correlation Matrix

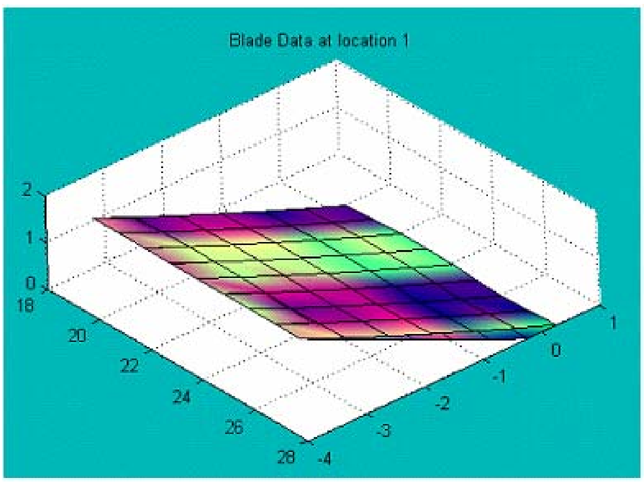

Figure 4 Given Blade Variation (sample #1)

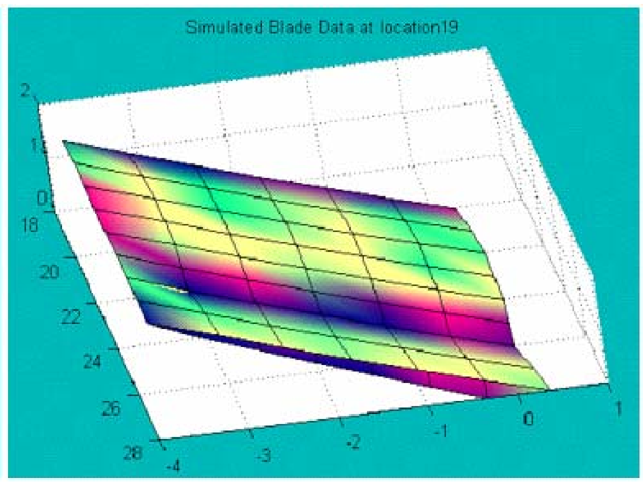

Figure 5 Simulated Blade Variation (sample # 19)

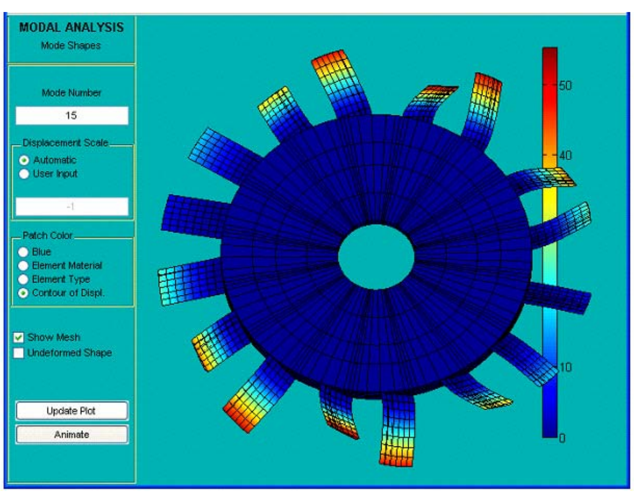

Figure 6 Mode # 15: Tuned Mode Shape

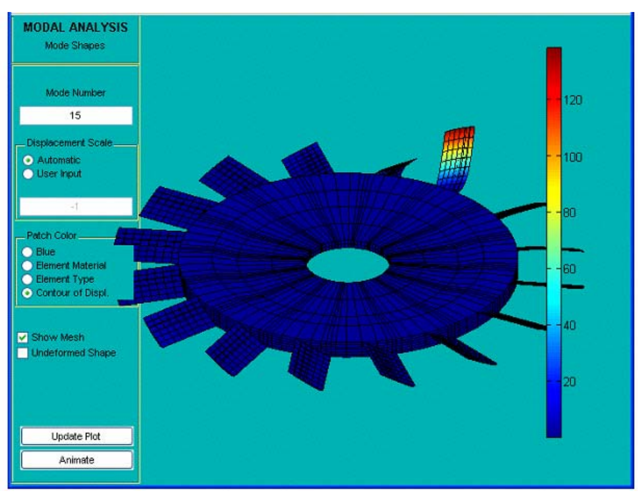

Figure 7 Mode #15: Mistuned Mode Shape

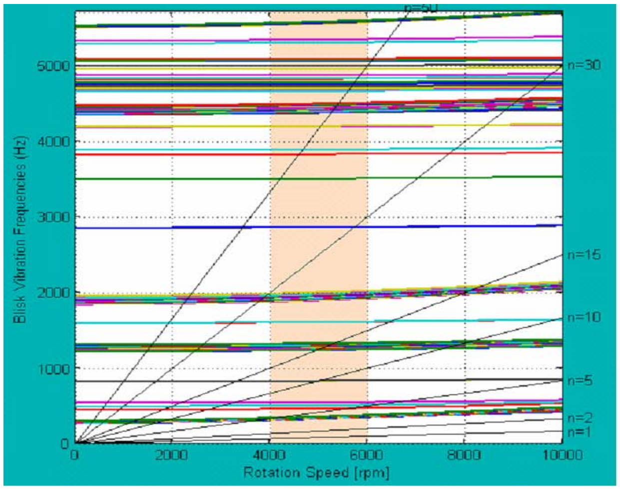

Figure 8 Mistuned Campbell Plot

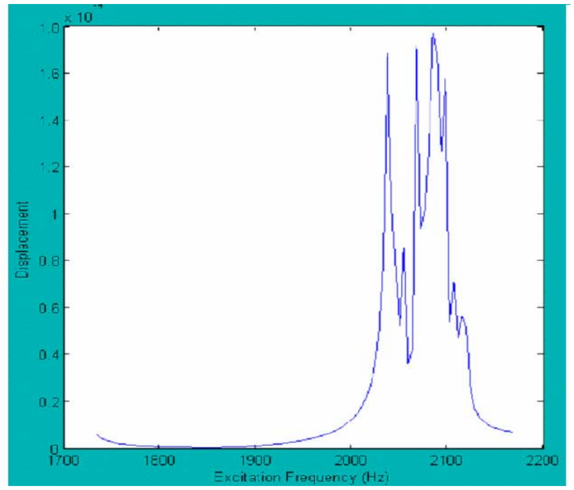

Figure 9 FRF for Node # 26, EO #13

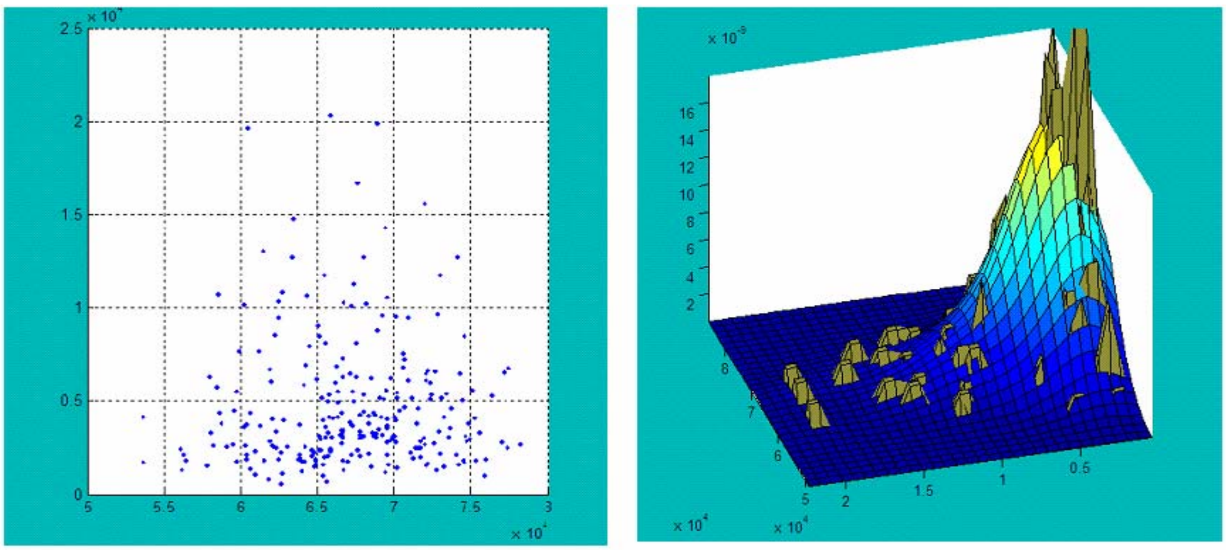

Figure 10 Computed Stress Data (Steady and Vibe Stresses) and Stress Bivariate Histogram and PDF

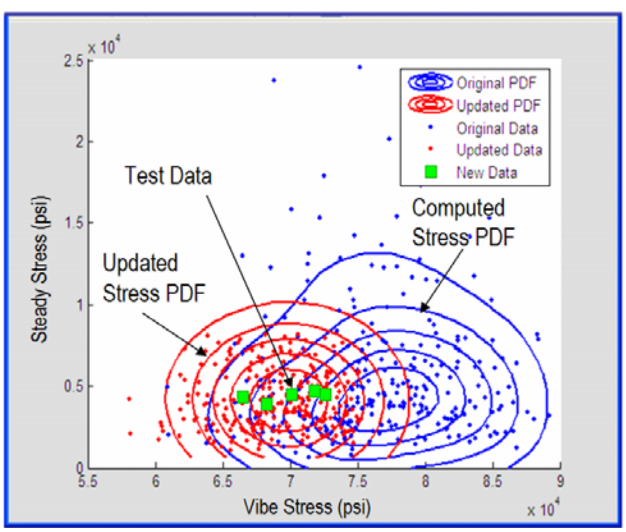

Figure 11 Prior and Updated Stress Bivariate PDF

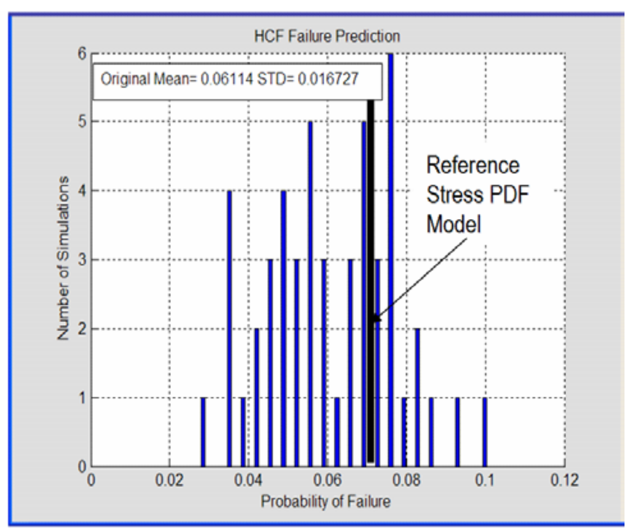

Figure 12 HCF Risks Including Lack of Data Effect

